Math Circles - Problem Set 2
Sequences and Series cont.

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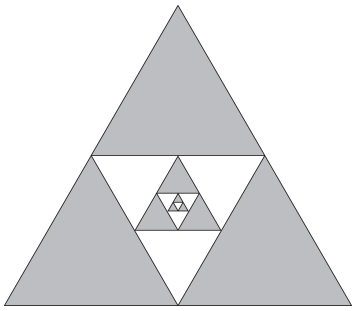
1. (a) Find the 5^{th} term in a geometric sequence with first term $a_1 = 432$ and common ratio $r = -2/3$.

(b) The 7^{th} term in a geometric sequence is 864, and the 12^{th} term is 6561. What is the 4^{th} term?

2. Each week grandma triples the number of raisins in her cookies. In the first week, she uses only 4 raisins. In total, how many raisins are used in the first 13 weeks?

3. The largest triangle in the figure below is equilateral with area 1. It is decomposed into infinitely many smaller equilateral triangles as shown.



Find the total area of the shaded triangles.

4. (a) Prove that $0.99999\ldots = 0.\overline{9}$ is equal to 1.

(b) Use a geometric series to write $1.1272727\ldots = 1.1\overline{27}$ as a fraction.

(c) Use a geometric series to write $0.\overline{632019}$ as a fraction.

5. When light hits a certain pane of glass, the glass reflects one half of the light, absorbs one fourth of the light, and transmits one fourth. A window is made of two panes of this glass separated by a small gap. If light of intensity I shines directly onto the window, what fraction is transmitted to the other side of the double pane?

6. Consider the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$.

(a) Find $\displaystyle\sum_{n=1}^{10}\left(\frac{1}{n}-\frac{1}{n+2}\right)$, the sum of the first 10 terms in the series.

(b) Find $\displaystyle\sum_{n=1}^{m}\left(\frac{1}{n}-\frac{1}{n+2}\right)$, the sum of the first m terms in the series.

(c) Compute the exact value of the infinite series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$.

Problems from Lesson 1

1. (a) If the sequence $7, a, b, 43, \ldots$ is arithmetic, what are the values of a and b?

(b) The 6^{th} term of an arithmetic sequence is 59, and the 21^{st} term is 14. What is the common difference?

2. The sum of the first n terms of a sequence is $n(n+1)(n+2)$.

(a) Write down the first 5 terms in this sequence.

(b) What is the 180^{th} term?

(c) Find an expression for the n^{th} term in the sequence.

3. (a) The sum of 100 consecutive integers is 9350. What is the largest of these integers?

(b) The sum of the first 6 terms in an arithmetic sequence is -81, and the sum of the first 40 terms is 4220. What is the 14^{th} term in the sequence?

4. (a) Find the sum of the first 1000 positive integers.

(b) Find the sum of the numbers between 1 and 1000 (including 1 and 1000) that are not multiples of 3.

(c) Determine the value of $1-2+3-4+\cdots+99-100$.

5. The numbers $2, 5, 8, 11, 14, \ldots$ are written in order in a book, beginning on page 1. There are 100 numbers on each page. On what page can the number 11 111 be found?

6. (a) The 3^{rd} term in a geometric sequence is 8 and the 6^{th} term is 17 576. What is the common ratio?

(b) The 10^{th} term of a geometric sequence is -6655 and the 13^{th} term is 5. What is the common ratio?

7. (a) Consider the recursive sequence defined by $a_1 = 9$ and $a_n = a_{n-1} - 4$ for all $n \geq 2$. Find a formula for a_n that depends only on n.

(b) Consider the recursive sequence defined by $a_1 = 1$, $a_2 = -1$, and $a_n = \left(\dfrac{n-3}{n-1}\right) a_{n-2}$ for $n \geq 3$. Determine the values of a_{2019} and a_{2020}.

8. Can a sequence with infinitely many terms be both arithmetic and geometric? If so, describe all sequences with this property.