# Math Circles - Problem Set 1
# Introduction to Linear Diophantine Equations

Zack Cramer - zcramer@uwaterloo.ca

February 28, 2018

**1.** Use the Euclidean algorithm to compute the following GCDs:

(a) $\gcd(204, 99)$

(b) $\gcd(1\,053, 993)$

(c) $\gcd(7\,404, 7\,032)$

**2.** Find a solution for each of the following LDEs, or explain why one does not exist.

(a) $204x + 99y = 3$

(b) $1\,053x + 993y = 7$

(c) $7\,404x + 7\,032y = 36$

**3.** Can 10 000 be expressed as a sum of two integers, one of which is divisible by 126 and the other divisible by 81? If so, find examples of such integers. If not, explain why.

**4.** Can 10 000 be expressed as a sum of two integers, one of which is divisible by 614 and the other divisible by 72? If so, find examples of such integers. If not, explain why.

**5.** Find an integer $n$, which, when divided by 78 leaves a remainder of 37; and when divided by 29 leaves a remainder of 17.

**6.** (a) Use the Euclidean algorithm to find a solution to $25x + 10y = 215$, the LDE from example (III).

(b) Does your answer in part (a) make sense in the context of the problem? If not, how can we find a solution that does make sense?

## Challenge Problems

**7.** (a) Use the division algorithm to show that $\gcd(k+1, k) = 1$ for any integer $k$.

(b) Use the division algorithm twice (i.e., the Euclidean algorithm) to show that

$$\gcd(7k+6, 6k+1) = 1$$

for any integer $k \geq 1$.

**8.** (a) Let $a$ and $b$ be integers. For which integers $c$ does $ax + by = c$ have a solution?

(b) Let $a, b$, and $c$ be integers. For which integers $d$ does $ax + by + cz = d$ have a solution?

(c) Find a solution to the 3-variable LDE $18x + 14y + 63z = 5$.