Implicit Differentiation (Not in textbook)

We are now masters at finding y' when $y = f(x)$ (explicit function)

But what if we can't solve for y explicitly??

Ex: (1) $x^2 + y^2 = 1$

(2) $2y + y^{23} - x^3 = 20$

(3) $3x^3y^3 + x^2y + 13x = 12e^x$

Taking derivatives of expressions like these is called implicit differentiation and is really just chain rule!

Remember: y is a function of x.

Ex: What's y' when $x^2 + y^2 = 1$?

Solution: Differentiate everything (Don't forget to use chain rule on y terms)

$$x^2 + y^2 = 1 \Rightarrow 2x + 2y \cdot y' = 0$$

$$\Rightarrow 2y \cdot y' = -2x$$

$$\Rightarrow y' = \frac{-\cancel{2}x}{\cancel{2}y} = \boxed{\frac{-x}{y}}$$

Finish by solving for y'.

Ex: What's y' when $2y + y^{23} - x^3 = 20$?

Solution: Take derivatives everywhere!

$$2y + y^{23} - x^3 = 20 \quad \Rightarrow \quad 2y' + 23y^{22} \cdot y' - 3x^2 = 0$$

$$\Rightarrow \quad y'(2 + 23y^{22}) = 3x^2$$

$$\Rightarrow \quad y' = \boxed{\frac{3x^2}{2 + 23y^{22}}}$$

Ex: What's y' when $3x^3y^3 + x^2y + 13x = 12e^x$?

Solution: Derivatives! We'll need product rule...

$$3(3x^2 \cdot y^3 + x^3 \cdot 3y^2 \cdot y') + (2x \cdot y + x^2 \cdot y') + 13 = 12e^x$$

$$\Rightarrow 9x^2y^3 + 9x^3y^2 \cdot y' + 2xy + x^2 \cdot y' + 13 = 12e^x$$

$$\Rightarrow y'(9x^3y^2 + x^2) = 12e^x - 13 - 2xy - 9x^2y^3$$

$$\Rightarrow \quad y' = \boxed{\frac{12e^x - 13 - 2xy - 9x^2y^3}{9x^3y^2 + x^2}}$$