Chapter 6 - Differentiation Techniques

Goal: Develop faster methods for computing $f'(x)$ (since, ya know, using the definition sucks!)

§6.1 - Preliminary Methods

The Constant Rule: If $f(x) = k$ (k = a constant), then $f'(x) = 0$

To see why, use the definition:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{k-k}{h} = 0$$

VOILA!

Ex: $f(x) = \sqrt{3} \Rightarrow f'(x) = 0$, $g(x) = \pi + 7 \Rightarrow g'(x) = 0$.

What about powers of x?

In §5.4 we saw that
$f(x) = x \Rightarrow f'(x) = 1$
$f(x) = x^2 \Rightarrow f'(x) = 2x$
$f(x) = x^3 \Rightarrow f'(x) = 3x^2$.

The Power Rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

(Proof in text)

Ex: (1) $f(x) = x^4 \Rightarrow f'(x) = \boxed{4x^3}$

This was our guess back in §5.4!

(2) $f(x) = x^{11} \Rightarrow f'(x) = \boxed{11x^{10}}$.

(3) $f(x) = \sqrt{x}$ $(= x^{1/2}) \Rightarrow f'(x) = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$.

(4) $f(x) = \frac{1}{x^3}$ $(= x^{-3}) \Rightarrow f'(x) = -3x^{-4} = \boxed{\frac{-3}{x^4}}$.

(5) $f(x) = x^{\pi} \Rightarrow f'(x) = \boxed{\pi x^{\pi - 1}}$

(6) $f(x) = 8x^2 \Rightarrow f'(x) = ??$ What do we do when a constant is out front?

$\boxed{\text{The Coefficient Rule: The derivative of } K \cdot f(x) \text{ is } K \cdot f'(x)}$

(i.e., just ignore the coefficient!)

Ex: (1) $f(x) = 8x^2 \Rightarrow f'(x) = 8(2x) = \boxed{16x}$

(2) $f(x) = \frac{6}{x}$ $(= 6x^{-1}) \Rightarrow f'(x) = -6x^{-2} = \boxed{\frac{-6}{x^2}}$

(3) $f(x) = 7\sqrt[3]{x^4}$ $(= 7x^{4/3}) \Rightarrow f'(x) = 7(4/3)x^{4/3 - 1}$
$= \boxed{\frac{28}{3} x^{1/3}}$

(4) $f(x) = \pi x^e \Rightarrow f'(x) = \boxed{\pi e x^{e-1}}$

(5) $f(x) = 2x - 3x^5 \Rightarrow f'(x) = ??$ How do we handle sums/differences?

$\boxed{\text{The Sum/Difference Rule: The derivative of } f(x) \pm g(x) \text{ is } f'(x) \pm g'(x).}$

(i.e., we do the derivative term-by-term)

Ex: (1) $f(x) = 2x - 3x^5 \Rightarrow f'(x) = \boxed{2 - 15x^4}$

(2) $f(x) = 12x^4 + 6\sqrt{x} - 1 \Rightarrow f'(x) = \boxed{48x^3 + 3x^{-1/2}}$

(3) $f(x) = \dfrac{x^3 + x^2}{x}$

We first rewrite $f(x) = \dfrac{x^3}{x} + \dfrac{x^2}{x} = x^2 + x.$

Now $f'(x) = \boxed{2x + 1}$

(4) $f(x) = (x-2)^3$

We don't Know how to handle products (yet!) so let's first expand:

$f(x) = (x-2)^3 = x^3 - 6x^2 + 12x - 8.$

Thus, $f'(x) = \boxed{3x^2 - 12x + 12}$

Ex: Find the values of x at which the tangent line to $f(x) = x^3 - x^2$ is horizontal

Solution: Horizontal tangent means $f'(x) = 0$.

$f'(x) = 3x^2 - 2x = x(3x-2) = 0.$

$\Rightarrow$ $\boxed{x = 0}$ or $3x - 2 = 0$, so $x = \boxed{\dfrac{2}{3}}$