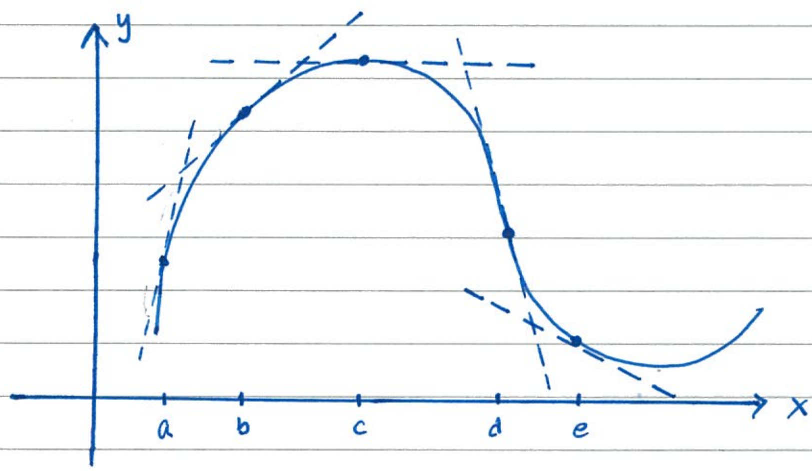
§5.5 - Graphical Differentiation.

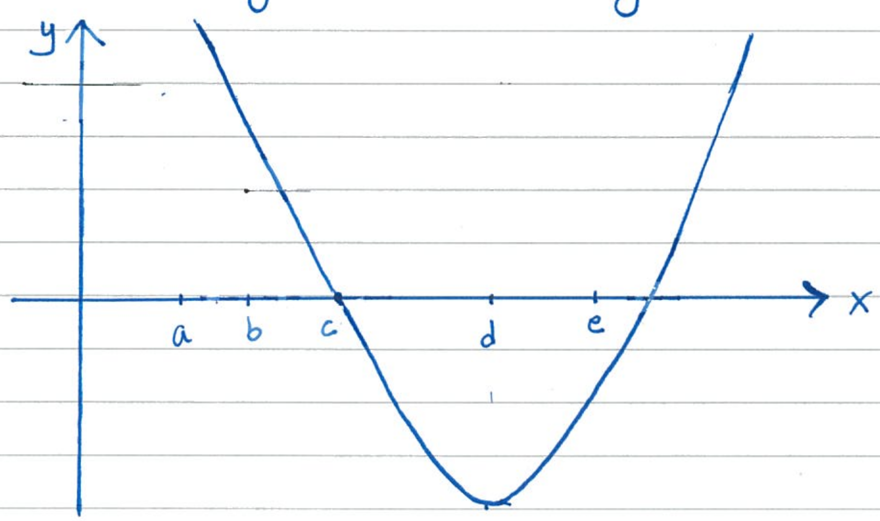
Say we know the graph of $f(x)$ but not necessarily its equation. We can still get information about $f'(x)$.

<u>Remember</u>: $f'(x)$ = slope of tangent line.



- At $x = a$, $f'(x)$ is very positive (steep slope up)
- At $x = b$, $f'(x)$ is positive, but less so than $f'(a)$.
- At $x = c$, $f'(x) = 0$ (horizontal tangent)
- At $x = d$, $f'(x)$ is very negative (steep slope down)
- At $x = e$, $f'(x)$ is negative, but less so than $f'(d)$.

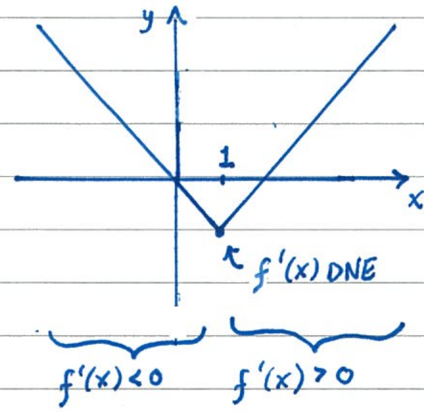
So $f'(x)$ may look something like

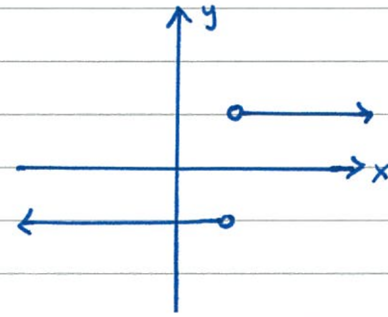


Ex: Sketch the graph of $f'(x)$ given the graph of $f(x)$:

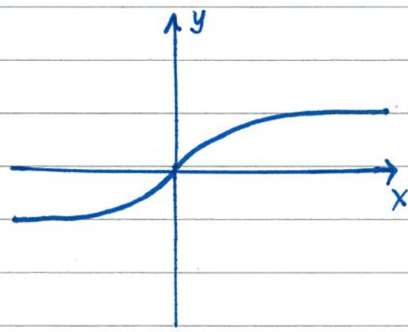
$f(x)$	$f'(x)$

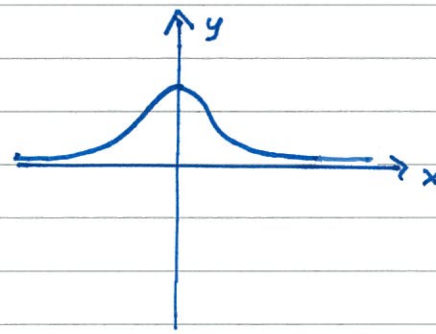
(1)





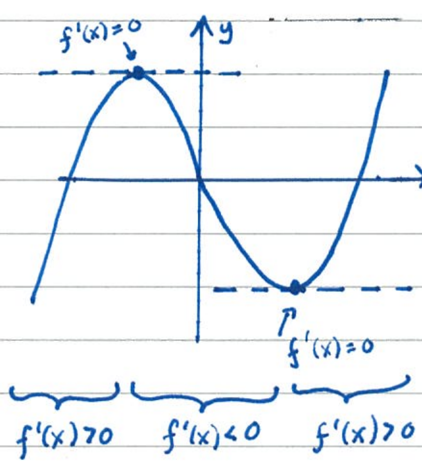
(2)

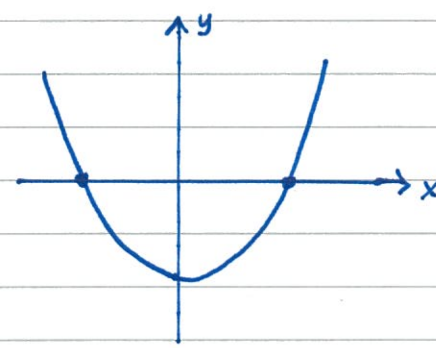




$f'(x)$ always positive,
tends to 0 on left/right.

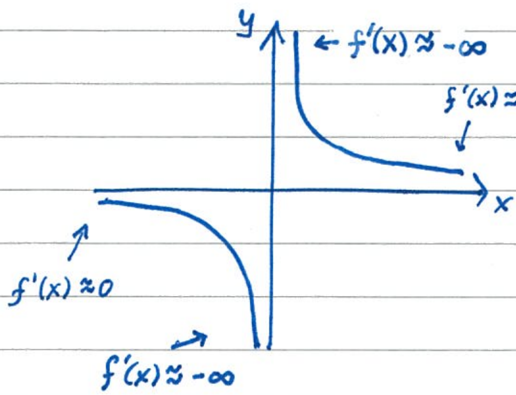
(3)

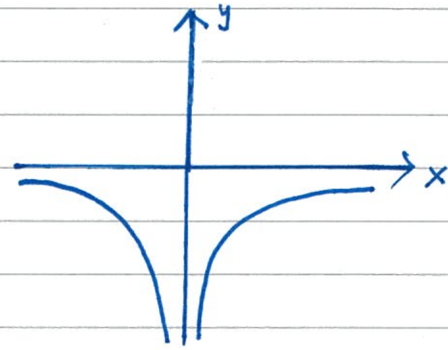




Note: Finding the points at which the tangent has slope 0 is often a good place to start. These are the x-intercepts for $f'(x)$!

(4) $f(x) = \frac{1}{x}$ $f'(x)$

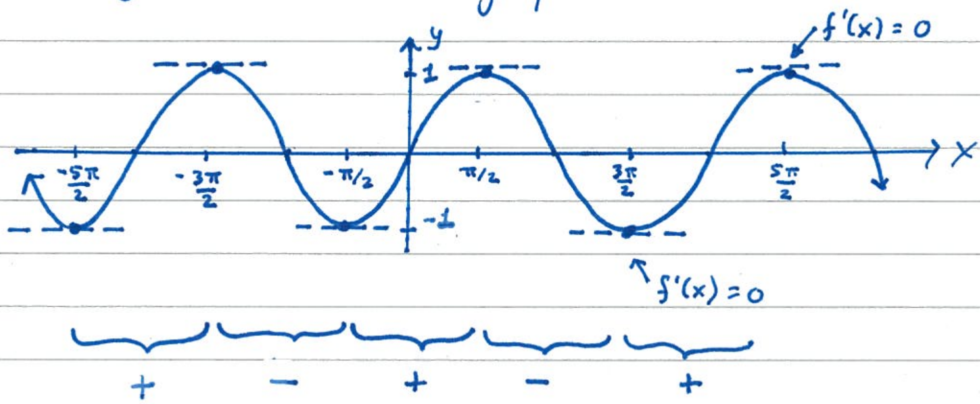




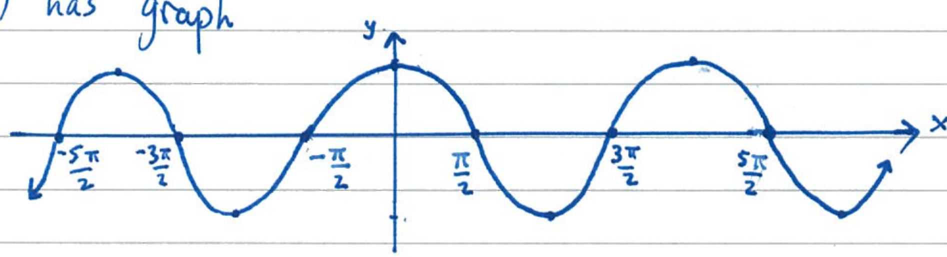
Note: $f'(x)$ is always < 0.

Ex: Sketch $f'(x)$ when $f(x) = \sin x$.

Solution: $f(x) = \sin x$ has graph



$f'(x)$ has graph

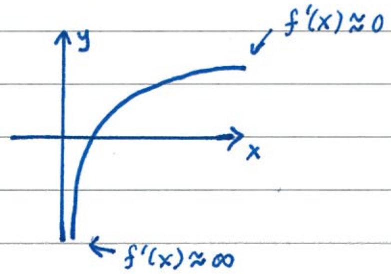


Wait... this graph looks familiar. It's $\cos x$! wow!!
We'll show later (by using the definition) that $f'(x) = \cos x$.

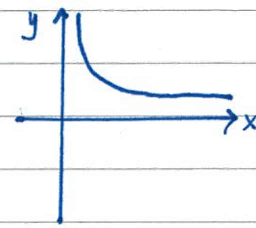
Ex: Sketch $f'(x)$ when $f(x) = \ln x$.

Solution

$f(x)$



$f'(x)$

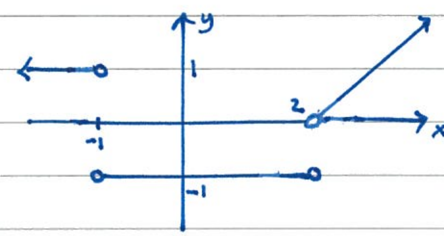


Note: $f'(x)$ is always > 0.

Again, the graph of $f'(x)$ looks familar! It's $\frac{1}{x}$!

Going backwards, we can sketch $f(x)$ given $f'(x)$
(sort of... there will be several possible functions $f(x)$.)

Ex: Sketch $f(x)$ given $f'(x)$:



Solution:
- $f(x)$ has slope 1 on $(-\infty, -1)$
- $f(x)$ has slope -1 on $(-1, 2)$
- Slope of $f(x)$ increases from 0 to infinity on $(2, \infty)$

2 possible solutions:
(there are many more)