Chapter 4 - Exponential, Logarithmic, Trigonometric Function

§4.1 - Exponential Functions.

An exponential function has the form

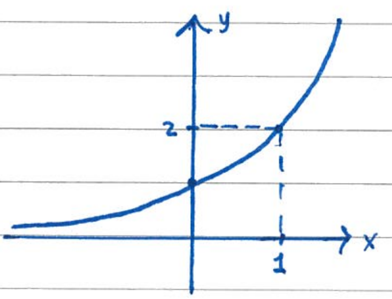
$$f(x) = a^x$$

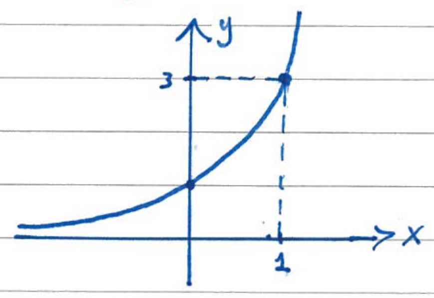
for some constant $a > 0$, $a \neq 1$

↑ otherwise $f(x) = 1$ always (BORING!)

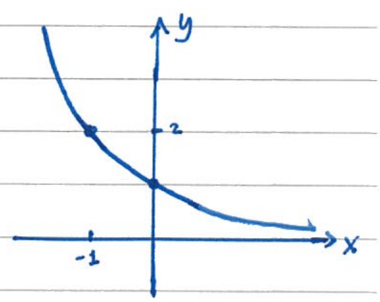
Exponential functions have
- domain $= \mathbb{R}$
- range $= (0, \infty)$
- horizontal asymptote at $y = 0$

Ex: $f(x) = 2^x$ $\qquad$ $g(x) = 3^x$





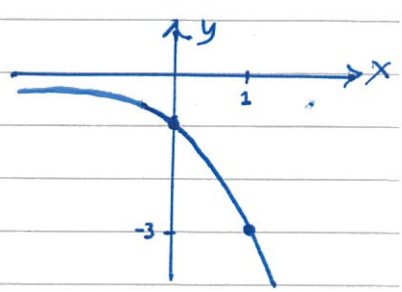
What about $h(x) = (\frac{1}{2})^x$?



<u>Note:</u> $h(x) = (2^{-1})^x = 2^{-x} = f(-x)$

so $h(x)$ is $f(x)$ reflected over y-axis.

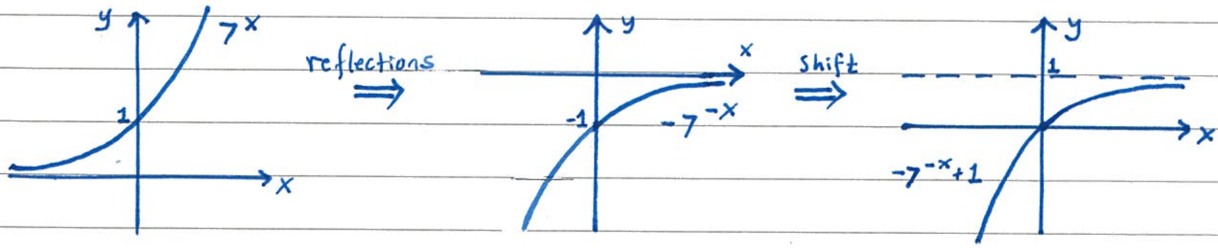
What about $k(x) = -3^x$?



<u>Note:</u> $k(x) = -g(x)$, so $k(x)$ is the reflection of $g(x)$ over x-axis:

Ex: Sketch the graph of $g(x) = -7^{-x} + 1$.

Solution: $g(x)$ is a reflection of $f(x) = 7^x$ over both axes, followed by an upward shift of 1 unit.



Solving Exponential Equations

When solving equations involving exponential functions, remember the following important rule!

$$\boxed{\text{For } a>0,\ a \neq 1, \quad a^x = a^y \Leftrightarrow x = y}$$

$\ast$ Bases must be the same!

Ex: Solve the following:

(1) $3^{7x} = 3^5$ (2) $3^{7x} = 9^{x+5}$

(3) $4^{x^2-1} = 8^{x^2-x}$ (4) $(1/7)^x = 49^{3x+1}$

Solution:

(1) $3^{7x} = 3^5 \Rightarrow 7x = 5 \Rightarrow \boxed{x = 5/7}$

(2) $3^{7x} = 9^{x+5}$ (Different bases ☹)

$\Rightarrow 3^{7x} = (3^2)^{x+5}$

$\Rightarrow 3^{7x} = 3^{2x+10}$ (Same base ☺)

$\Rightarrow 7x = 2x + 10$

$\Rightarrow 5x = 10$, so $\boxed{x = 2}$

(3) $4^{x^2-1} = 8^{x^2-x} \Rightarrow (2^2)^{x^2-1} = (2^3)^{x^2-x}$

$\Rightarrow 2(x^2-1) = 3(x^2-x)$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0$, so $\boxed{x=1}$ or $\boxed{x=2}$

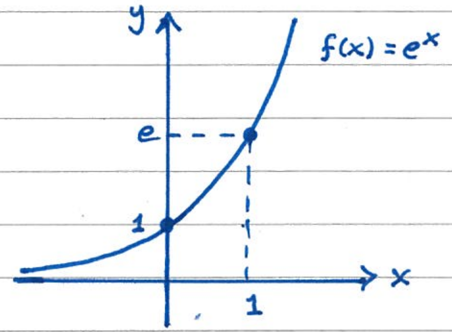
(4) $(1/7)^x = 49^{3x+1} \Rightarrow (7^{-1})^x = (7^2)^{3x+1}$

$\Rightarrow -x = 2(3x+1)$

$\Rightarrow 7x = -2$, so $\boxed{x = -2/7}$

What about things like $3^x = 5$?
We'll need logarithms! (§4.2)

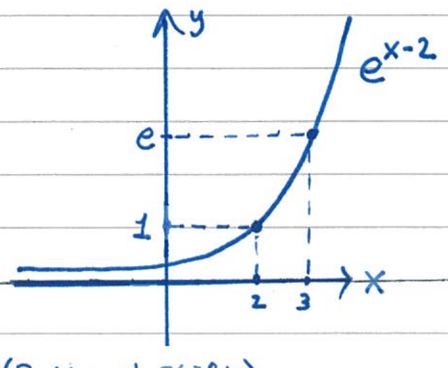
<u>The number "e"</u>



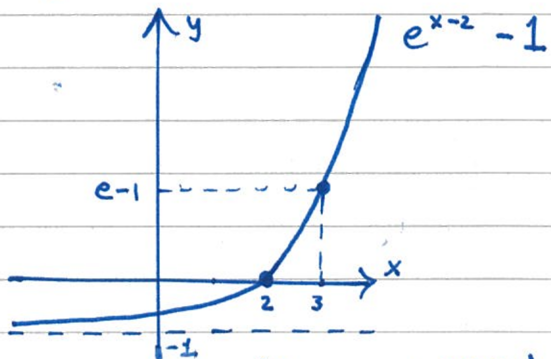
- $e \approx 2.71828$
- e is the unique number such that $f(x) = e^x$ has slope 1 at $x=0$ (important for Ch. 6!)
- Extremely important constant in math, econ, bio, finance, etc.

<u>Ex:</u> Sketch the graph of $f(x) = e^{x-2} - 1$.

<u>Solution:</u> Start with $y = e^x$ (shown above). Shift right by 2 units and down by 1 unit.



(Rightward Shift)



(Downward Shift)