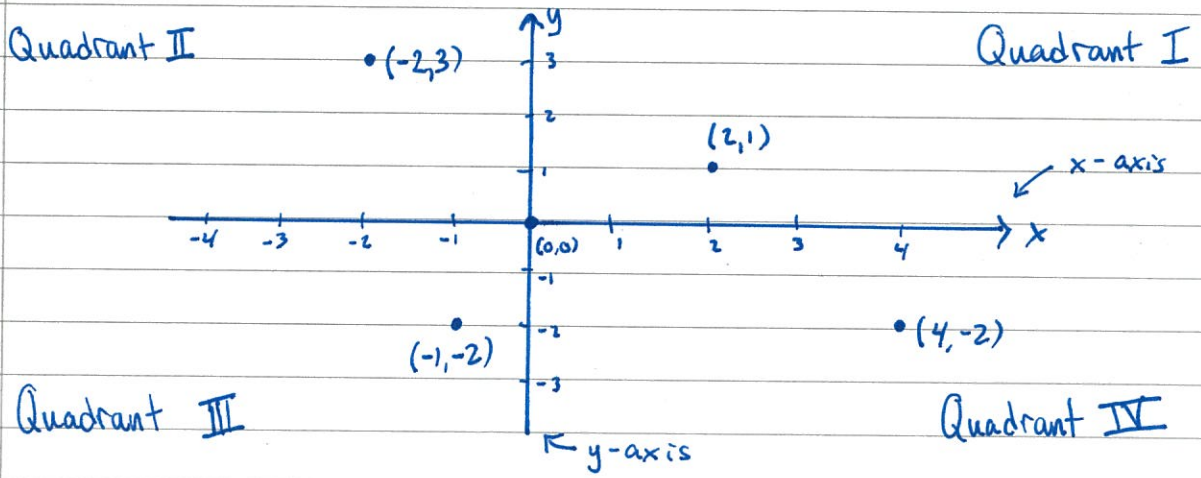
Chapter 3 - Functions

§3.1 - Lines and Linear Functions

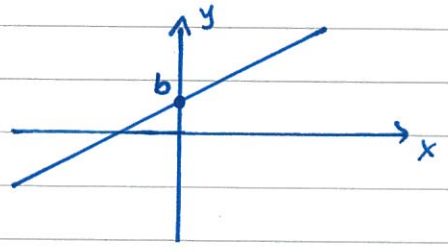
We will describe points and lines in Cartesian coordinates (x, y):

x-coordinate: how far left/right
y-coordinate: how far up/down.

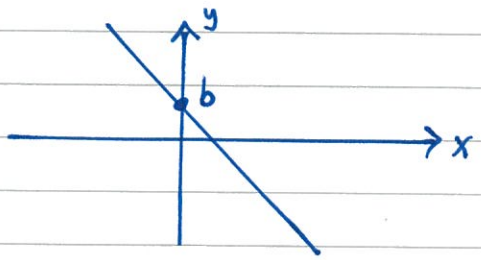


Lines: A line has equation $y = mx + b$

Here, m = slope of the line
b = y-intercept (where it crosses the y-axis)



Line with positive slope
$(m > 0)$



Line with negative slope
$(m < 0)$

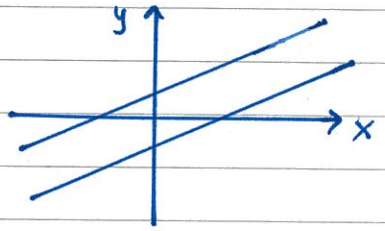
Slope = $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$

Given two points (x_1, y_1), (x_2, y_2) on the line, we can calculate the slope as

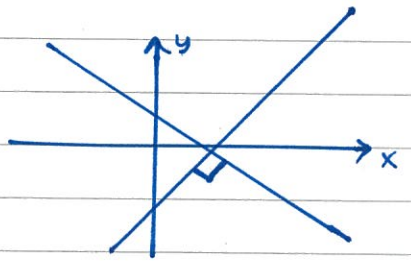
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Horizontal lines have slope $m = 0$
- Vertical lines have "infinite slope" (we say it is undefined)

Two lines are parallel if they have the same slope



Two lines are perpendicular if one has slope m and the other has slope $\frac{-1}{m}$. They meet at 90°.



What types of questions can we ask about lines?

(1) Find equation of a line through 2 given points

(2) Find equation of a line with slope m, passing through a given point (x_0, y_0)

(3) Find equation of a line through point (x_0, y_0) that is parallel or perpendicular to a given line.

Ex: Find the equation of the line through $(-1, 1)$ and $(1, 5)$.

Solution: Need 2 things: → slope m
→ y-intercept b.

Use the slope formula for m:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{1-(-1)} = \frac{4}{2} = \boxed{2}$$

So, the line is $y = 2x + b$.

To get b, plug in either point. We'll do $(1, 5)$.

$$y = 2x + b \Rightarrow 5 = 2(1) + b \Rightarrow b = 5 - 2 = \boxed{3}$$

Thus, our line is $\boxed{y = 2x + 3}$

Ex: Find the equation of the line through $(6, 1)$ and parallel to $y = \frac{4}{3}x - 1$.

Solution: Parallel lines have the same slope, so $\boxed{m = 4/3}$.
The y-intercept b may be different!

Plugging in $(6, 1)$ to our equation $y = 4/3\,x + b$, we get

$$6 = 4/3(1) + b \Rightarrow b = 6 - 4/3 = 18/3 - 4/3 = \boxed{14/3}.$$

So, our equation is $\boxed{y = \frac{4}{3}x + \frac{14}{3}}$

Ex: Find the equation of the line through $(14, -4)$ and perpendicular to $2y - 7x = 1$.

Solution: Hmm... What's the slope of $2y - 7x = 1$? Let's write it in the form $y = mx + b$.

$$2y - 7x = 1 \Rightarrow 2y = 7x + 1 \Rightarrow y = \frac{7}{2}x + \frac{1}{2}.$$

Aha! This line has slope $7/2$. Since ours is perpendicular, the slope is

$$m = \frac{-1}{(7/2)} = \boxed{\frac{-2}{7}}$$

Get b as before.

$$y = -2/7\,x + b \Rightarrow -4 = -2/7\,(14) + b$$
$$\Rightarrow -4 = -4 + b, \quad \text{so } b = \boxed{0}$$

Our line is $\boxed{y = -2/7\,x}$

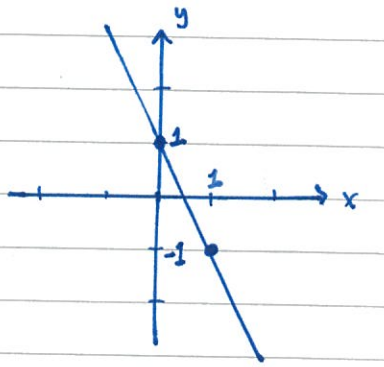
Graphing Lines

To graph a line, plot any 2 points on the line and connect them.

Ex: Graph $y = -2x + 1$.

Solution: The y-intercept is at $b = 1$ so it passes through $(0, 1)$.

At $x = 1$, $y = -2(1) + 1 \quad -1$, so it passes through $(1, -1)$.



Determining Where 2 Lines Meet:

To find where 2 lines intersect, equate their y values and solve for x.

Ex: Find the point at which $y = x+1$ intersects $y = -x+3$.

Solution: $x+1 = y = -x+3$, so we get

$$x+1 = -x+3 \Rightarrow 2x+1 = 3$$
$$\Rightarrow 2x = 2$$
$$\Rightarrow x = 1.$$

The lines cross when $x = 1$. Plug in this value into either line to get y.

$$y = x+1 \Rightarrow y = 1+1 = 2$$

The point of intersection is $\boxed{(1,2)}$.