TEST	**STATEMENT**	**NOTES**	**EXAMPLES**
Geometric Series Test	If $a, r \in \mathbb{R}$ with $a \neq 0$ then the geometric series $$\sum_{n=0}^{\infty} ar^n = \begin{cases} \dfrac{a}{1-r} & \text{if } \lvert r\rvert < 1, \\ \text{divergent} & \text{if } \lvert r\rvert \geq 1. \end{cases}$$	- The N^{th} partial sum is given by $$S_N = \frac{a(1-r^{N+1})}{1-r}.$$	$\sum_{n=0}^{\infty} \frac{(-3)^n}{4^n}$ $\sum_{n=1}^{\infty} \frac{2^{2n}}{5^{n+1}}$
Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ DNE, then $\sum_{n=1}^{\infty} a_n$ diverges.	- Often a good test to start with. - If $\lim_{n\to\infty} a_n = 0$, no conclusions can be made. (e.g., $\sum \frac{1}{n}$ diverges and $\sum \frac{1}{n^2}$ converges.)	$\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$ $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$
Integral Test	Suppose $f(x)$ is continuous, positive, and decreasing on $[1, \infty)$. (i) If $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges. (ii) If $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} f(n)$ diverges.	- Useful when $\int_1^{\infty} f(x)\,dx$ is easy to calculate. - When convergent, we have the remainder estimate $$\int_{N+1}^{\infty} f(x)\,dx \leq R_N \leq \int_N^{\infty} f(x)\,dx$$ where $R_N = S - S_N$ and S is the sum of the series.	$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$
p-Series Test	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$ and diverges when $p \leq 1$.	- Often used with comparison tests.	$\sum_{n=1}^{\infty} \frac{1}{(2n)^2}$ $\sum_{n=1}^{\infty} \frac{1}{n}$
Comparison Test	Suppose that $0 \leq a_n \leq b_n$ for all n sufficiently large. (i) If $\sum b_n$ converges, then $\sum a_n$ converges. (ii) If $\sum a_n$ diverges, then $\sum b_n$ diverges.	- No conclusions if $\sum b_n$ diverges or $\sum a_n$ converges.	$\sum_{n=1}^{\infty} \frac{n+2}{(n+1)^3}$ $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+n+3}}$

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Limit Comparison Test	Suppose that $\sum a_n$ and $\sum b_n$ are series of positive terms, and let $$L = \lim_{n\to\infty} \frac{a_n}{b_n}.$$ If L exists and $0 < L < \infty$, then $\sum a_n$ and $\sum b_n$ either both converge or both diverge.	- Usually works well with fractions involving polynomials, roots, or exponentials. - When applying this test to $\sum a_n$, we usually define b_n using only the most dominant parts of a_n. - No conclusions when $L = 0$ or $L = \infty$	$\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt[3]{5+n^7}}$ $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n+5^n}$
Alternating Series Test	Consider the series $$\sum_{n=1}^{\infty} (-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots,$$ where $b_n > 0$ for all n. If (i) $\{b_n\}$ is a decreasing sequence, and (ii) $\lim_{n\to\infty} b_n = 0$, then $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ converges.	- When convergent, we have the remainder estimate $$\lvert S - S_N \rvert \le b_{N+1},$$ where S is the sum of the series.	$\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$ $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{n!}}$
Ratio Test	Suppose that $L = \lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$ exists or is equal to ∞. (i) If $L < 1$, then $\sum a_n$ converges absolutely. (ii) If $L > 1$, then $\sum a_n$ diverges. (iii) If $L = 1$, the test is inconclusive.	- Useful when the terms of the series involve factorials.	$\sum_{n=1}^{\infty} \frac{10^n}{n \cdot 4^{2n+1}}$ $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,2^n}$ $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{3^n}$
Root Test	Suppose that $L = \lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert}$ exists or is equal to ∞. (i) If $L < 1$, then $\sum a_n$ converges absolutely. (ii) If $L > 1$, then $\sum a_n$ diverges. (iii) If $L = 1$, the test is inconclusive.	- Useful when terms of the series involve n^{th} powers.	$\sum_{n=1}^{\infty} \left(\tan^{-1} n\right)^n$ $\sum_{n=1}^{\infty} \frac{n^n}{n^3 e^n}$ $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^{2n}$