

§ 1.5 - Functions

We say that y is a function of x (written $y = f(x)$)

if there is a rule that associates exactly one

y -value to every x -value.

x is the independent variable

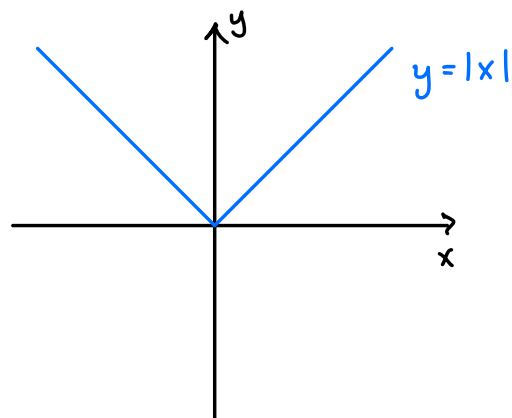
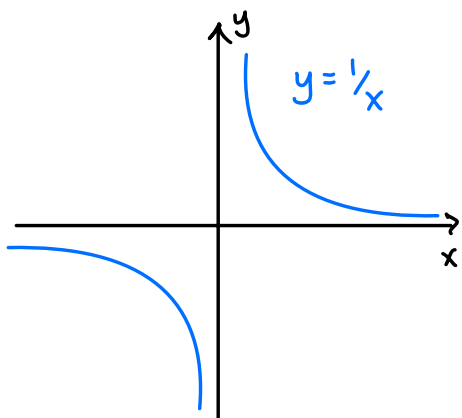
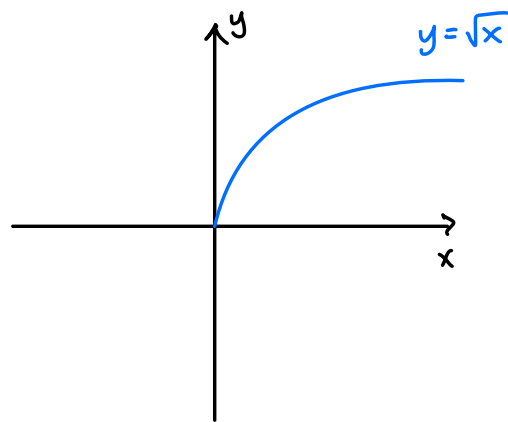
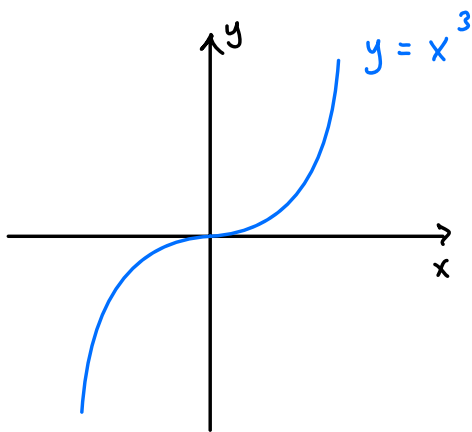
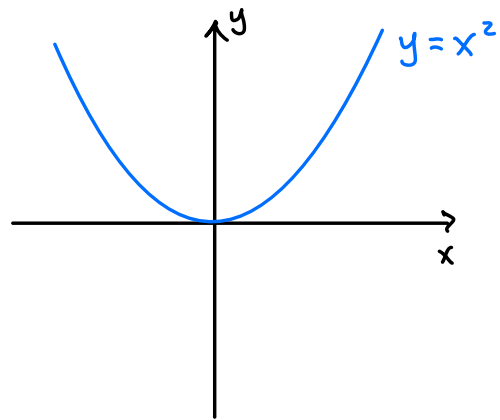
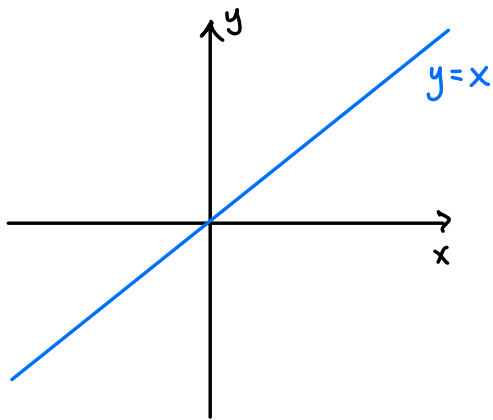
y is the dependent variable

Ex: $y = f(x) = x^3 - x$

e.g., $x = 1 \Rightarrow y = 1^3 - 1 = 0$

$x = -2 \Rightarrow y = (-2)^3 - (-2) = -8 + 2 = -6$

You're probably familiar with the following "basic" functions and their graphs...



and also with the following families of functions:

(i) Polynomials (of degree n)

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where n is a positive integer, $a_i \in \mathbb{R}$ and

$a_n \neq 0$.

The set of all real numbers,
also written as $(-\infty, \infty)$.

Ex: $f(x) = 2x^3 + x - 1$ is a polynomial of degree 3.

(ii) Rational functions, which have the form

$$f(x) = \frac{P(x)}{Q(x)}$$

where P, Q are polynomials.

Ex: $f(x) = \frac{2x^3 + x - 1}{x^5 + \frac{1}{2}}$ is a rational function.

Note: Not every equation we encounter will describe a function $y=f(x)$!

Ex: $x=y^2$

Here y isn't a function of x since most

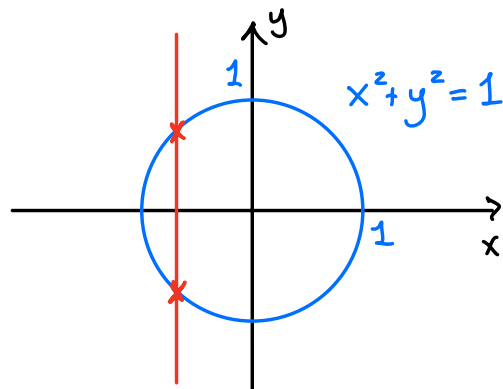
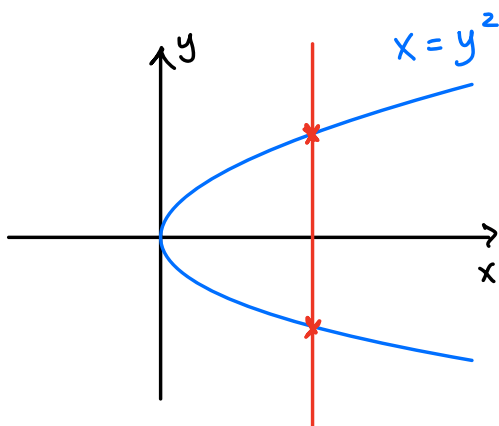
x 's produce two y -values: $y = \pm\sqrt{x}$

Ex: The unit circle $x^2+y^2=1$

Same issue: $y^2 = 1-x^2 \Rightarrow y = \pm\sqrt{1-x^2}$

\Rightarrow Not a function.

We can also see this graphically:



These curves fail the vertical line test and hence aren't graphs of functions $y = f(x)$.

Vertical Line Test: Any vertical line intersects the graph of a function $y = f(x)$ at most once.

You may see the function notation

$$f: D \longrightarrow \mathbb{R}.$$

Here, D is the domain — the set of all inputs x that can be plugged into f .

Possible Domain Issues:

- Division by 0
- Square root (or $\sqrt[n]{}$ where n is even) of something < 0 .
- Log of something ≤ 0 .

Ex: Find the domain of $f(x) = \sqrt{\frac{x-3}{x+2}}$.

Solution: Can't divide by 0, so $x \neq -2$.

Also need $\frac{x-3}{x+2} \geq 0$. We can look at the sign

of the numerator ($=0$ at $x=3$) and denominator

($=0$ when $x=-2$).

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	-2	3	
$x-3$	-	-	+
$x+2$	-	+	+
$\frac{x-3}{x+2}$	+	+	+

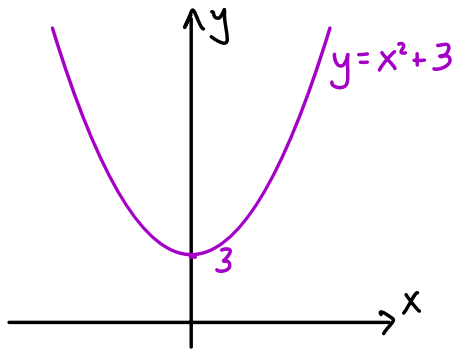
\therefore Domain is $(-\infty, -2) \cup [3, \infty)$, or equivalently

$$\{x \in \mathbb{R} : x < -2 \text{ or } x \geq 3\}$$

The range of f is the set of all outputs y that f can produce from the x 's in its domain.

Ex: $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2 + 3$ has range

$[3, \infty)$ since $y = x^2 + 3 \geq 0 + 3 \Rightarrow y \in [3, \infty)$



And every y in $[3, \infty)$
can indeed be produced!