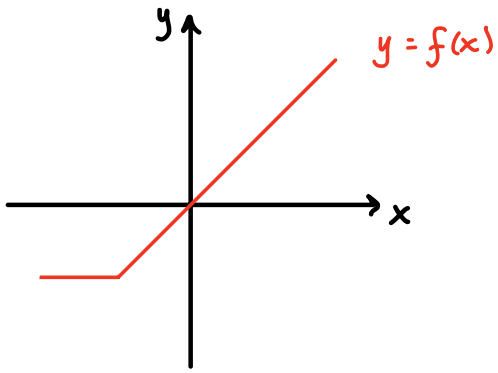
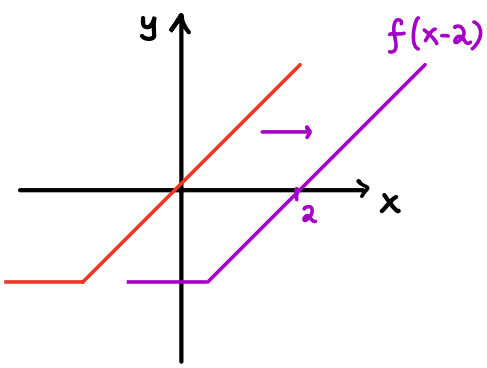
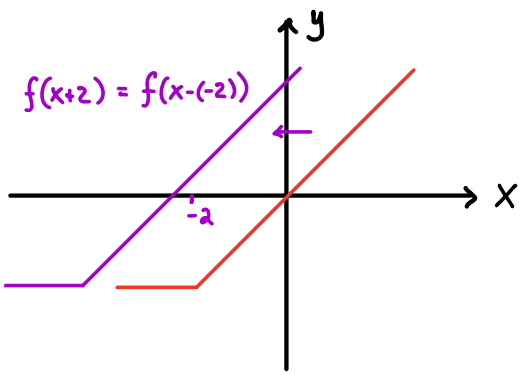
Graphing Functions Using Transformations

Starting with the graph of $y = f(x)$, we can apply transformations to graph many related functions!

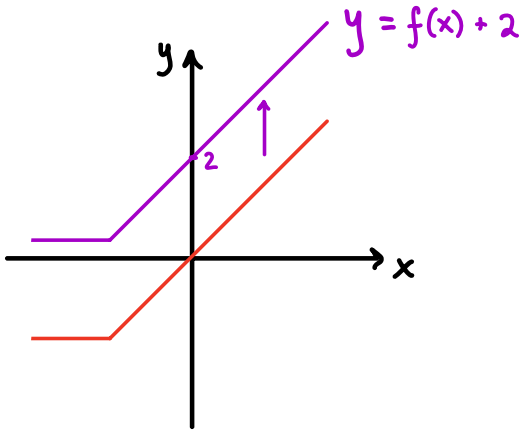


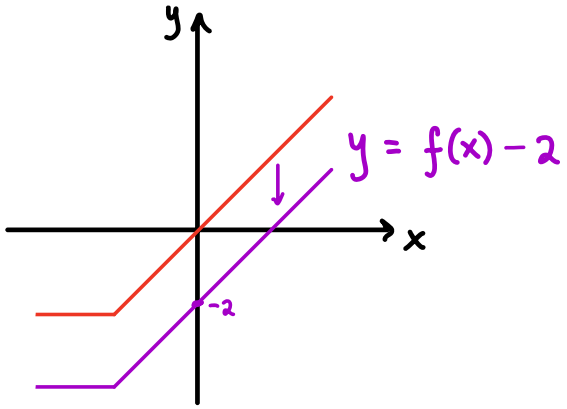
$y = f(x-a) \implies$ Shift by a units in the positive x direction





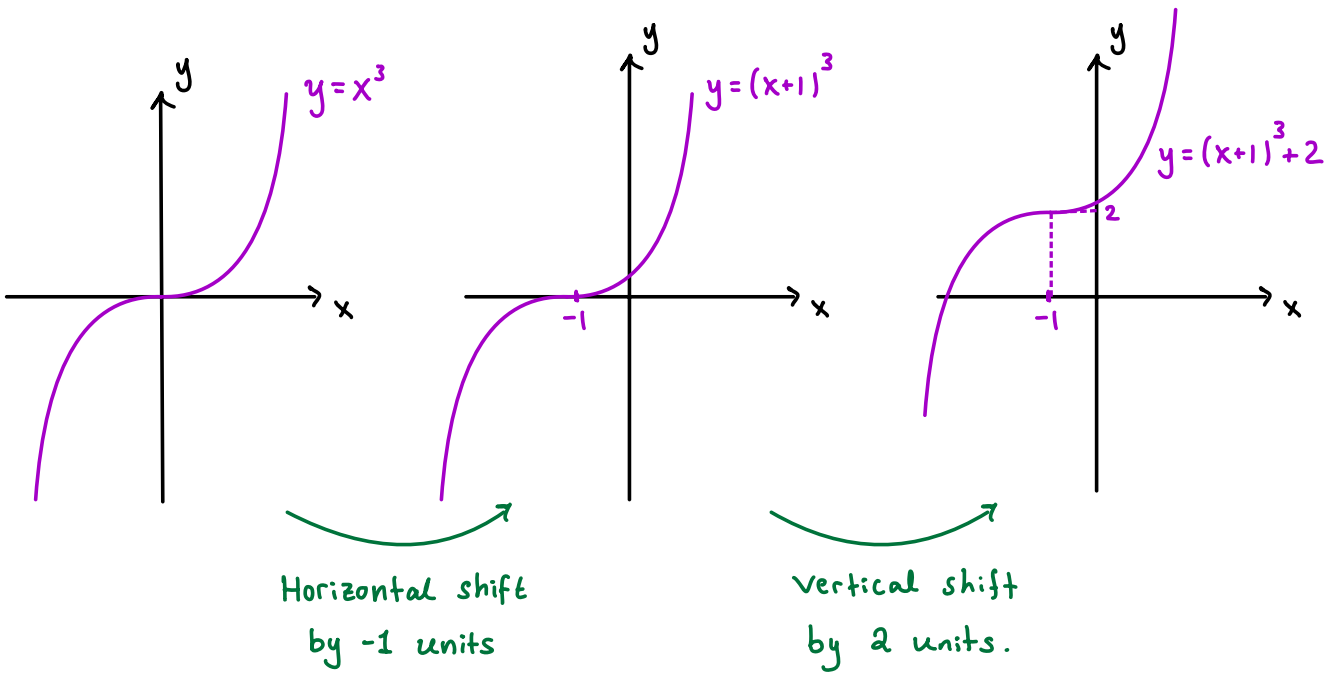
$y - a = f(x)$ (or $y = f(x) + a$) $\implies$ Shift by a units in the positive y direction





Ex: Starting with the graph of $y = x^3$, sketch the graph of $y = (x+1)^3 + 2$

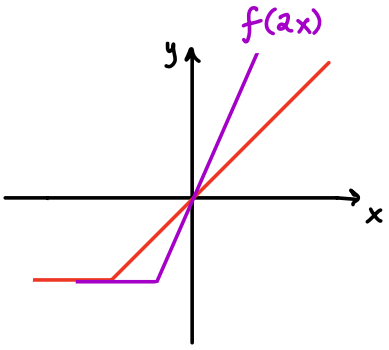
Solution



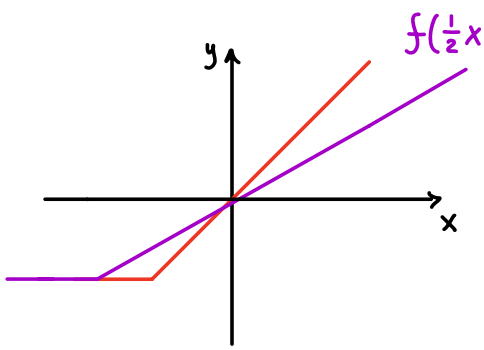
We can also stretch, compress or reflect a

graph about an axis.

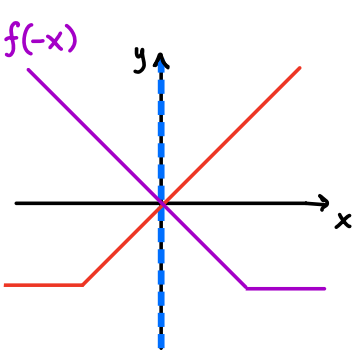
$y = f(kx) \Rightarrow$ horizontal stretch / compression (& reflection over y-axis if $k<0$)



(compression if $|k|>1$)

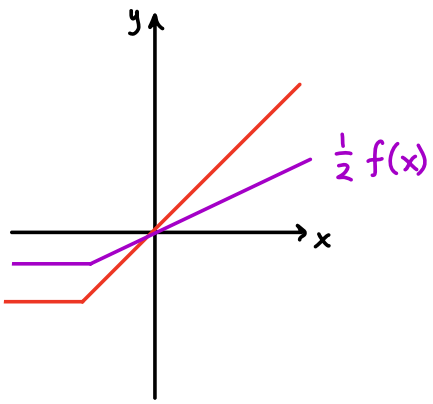


(stretch if $|k|<1$)

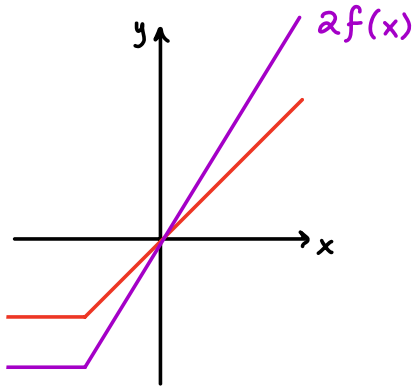


(reflection)

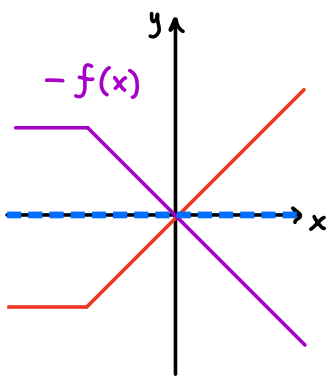
$y = k \cdot f(x) \Rightarrow$ vertical stretch / compression (& reflection over x-axis if $k<0$)



(compression if $|k|<1$)



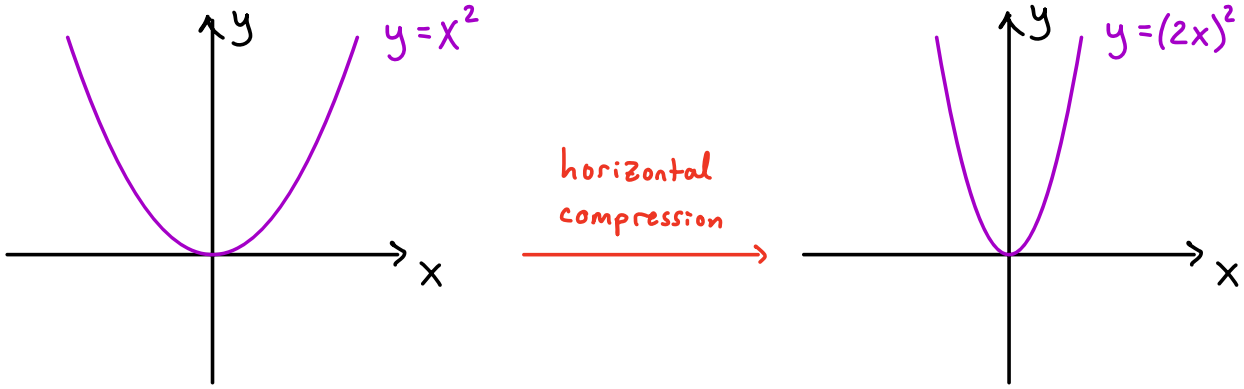
(stretch if $|k|>1$)

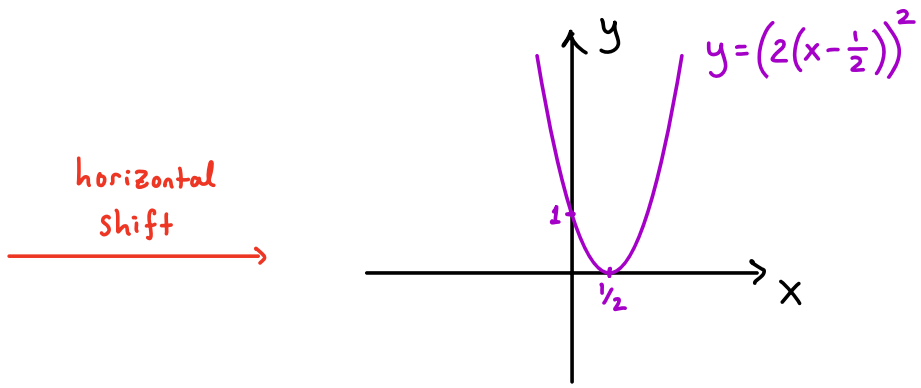


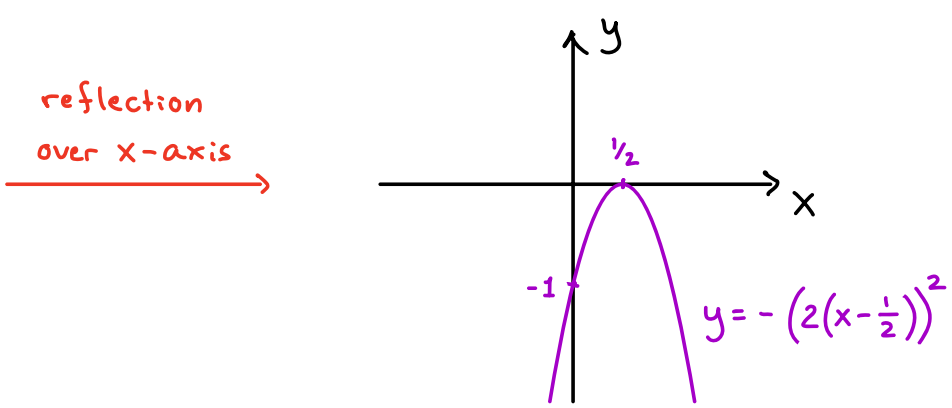
(reflection)

Ex: Sketch the graph of $y = 1 - (2x-1)^2$

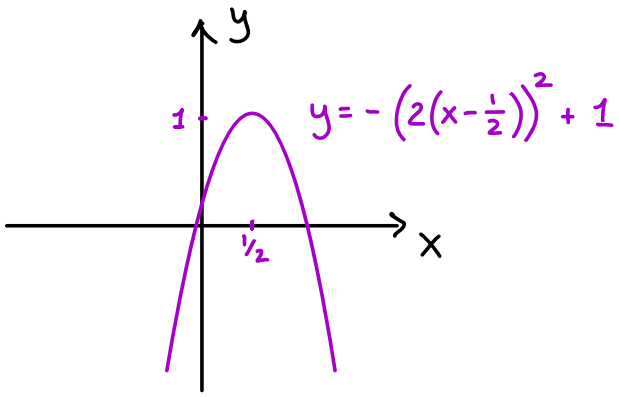
Solution: Let's rewrite as $y = -\left(2\left(x-\frac{1}{2}\right)\right)^2 + 1$ to better see the transformations.







Vertical shift →



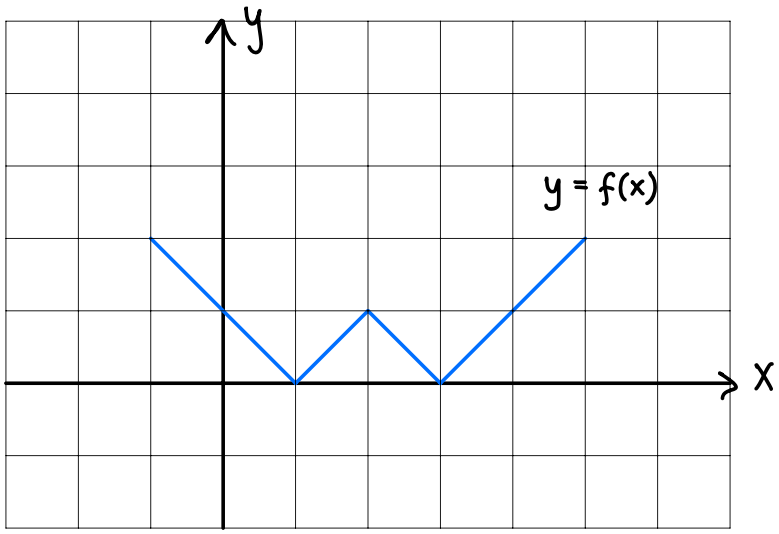
Remark: Be mindful of your order of operations!

Ex: For $f(x) = 1 - x^2$, we should first reflect $y = x^2$ over the x-axis and then shift up by 1 unit.

Would we obtain the same graph if we performed these transformations in the opposite order?

Additional Exercise:

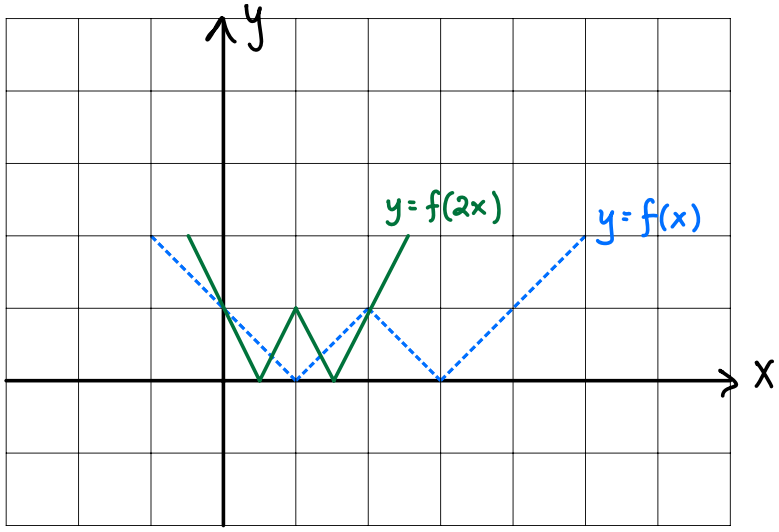
The graph of $y = f(x)$ is shown below.



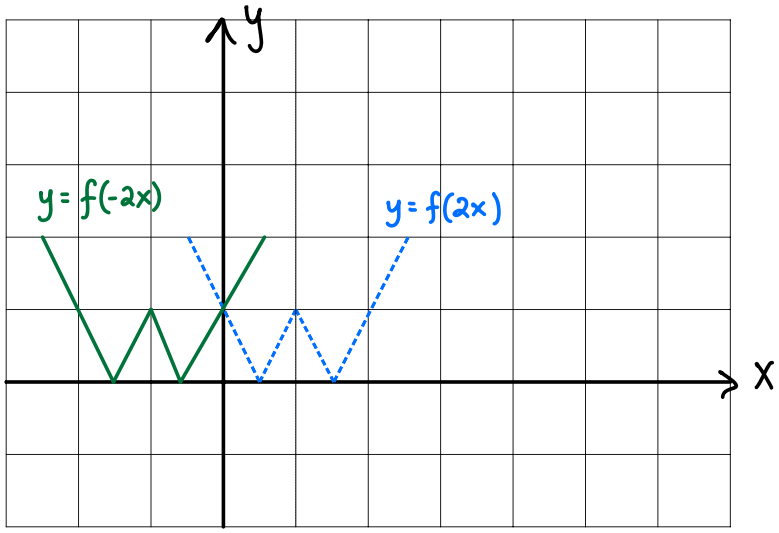
Sketch the graph of $y = 2f(-2x) - 1$

Solution: We'll apply the transformations one by one.

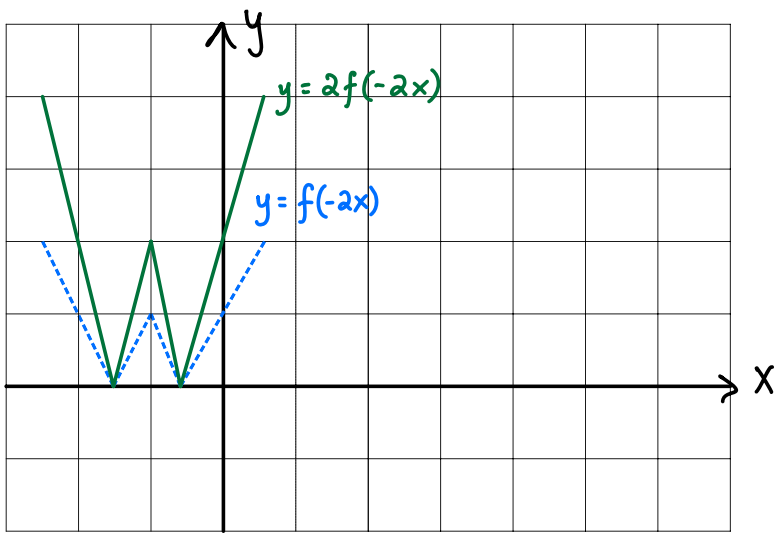
horizontal compression →



reflection over y-axis →



Vertical Stretch →



Vertical Shift →