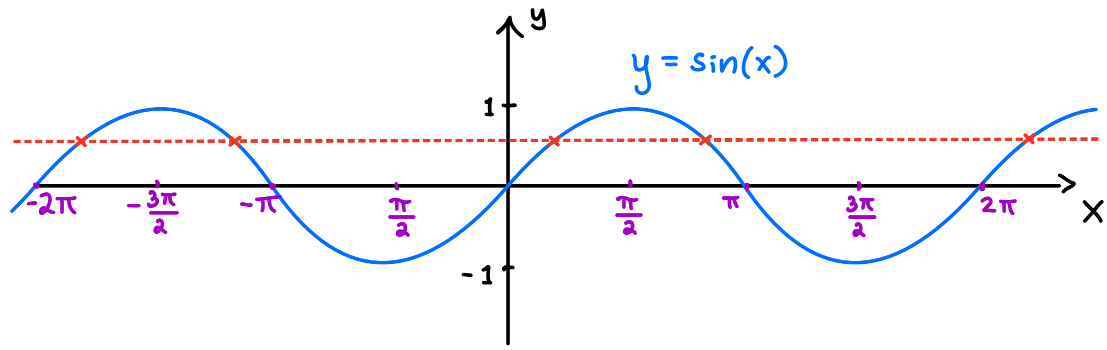
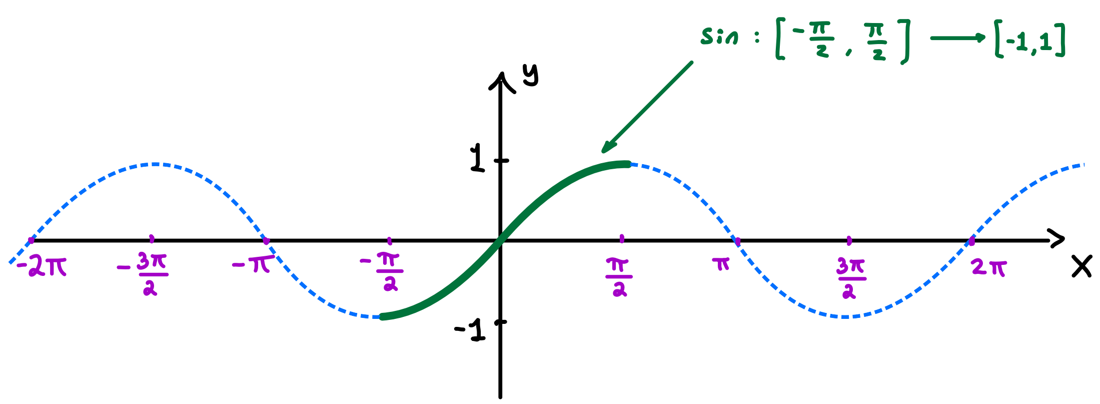
§1.8 - Inverse Trig Functions

Does $f(x) = \sin(x)$ have an inverse? <u>No!</u> It fails the horizontal line test (badly!)



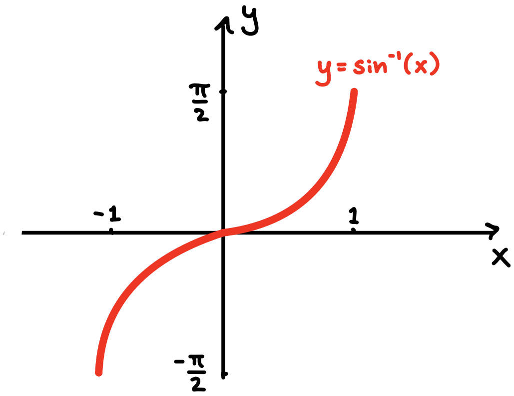
However, we can restrict $\sin(x)$ to a domain where it <u>does</u> pass the horizontal line test and hence has an inverse! We'll restrict to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



We then define

$\sin^{-1}: [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

to be the inverse function.



Note: $\sin^{-1}(x)$ is also denoted $\arcsin(x)$.

Examples: $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ $\left(\text{since } \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}\right)$

Wait... $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ too, so is it also correct to write $\sin^{-1}\left(\frac{1}{2}\right) = \frac{5\pi}{6}$? No!

Remember, $\sin^{-1}$ always outputs values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so it "chooses" $\frac{\pi}{6}$ over $\frac{5\pi}{6}$, $\frac{13\pi}{6}$, etc...

$\sin^{-1}(2)$ is undefined.

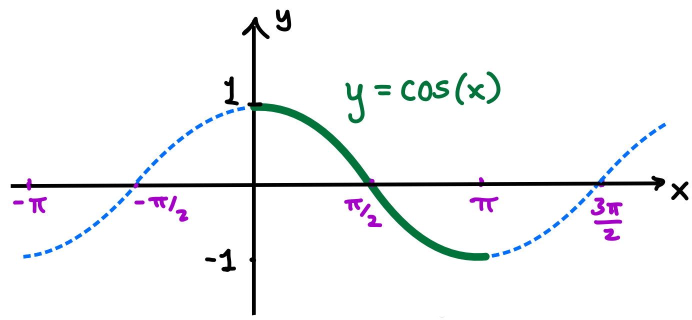
$\sin\left(\sin^{-1}(1)\right) = \sin\left(\frac{\pi}{2}\right) = 1$

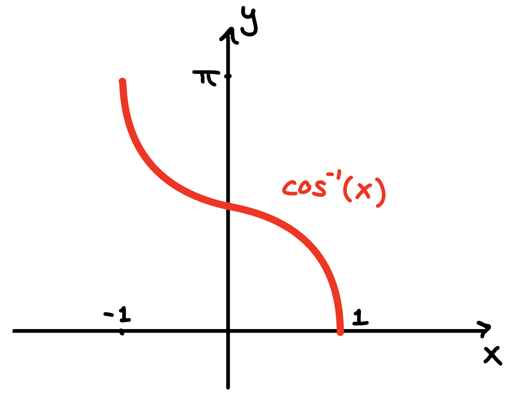
$\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

For $\cos(x)$, we define $\cos^{-1} : [0, \pi] \longrightarrow [-1, 1]$

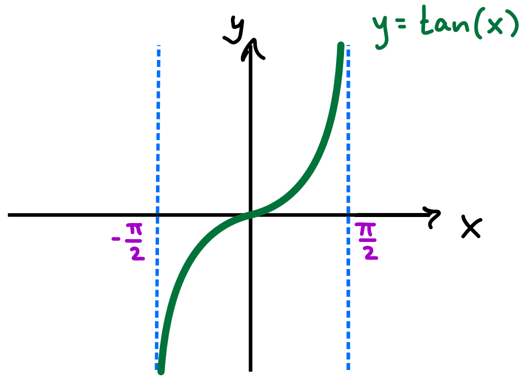
also known as arccos

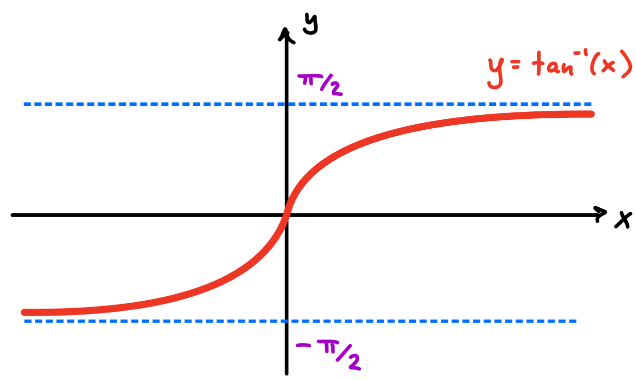




For $\tan(x)$, we define $\tan^{-1} : (-\infty, \infty) \longrightarrow (-\pi/2, \pi/2)$

also known as arctan





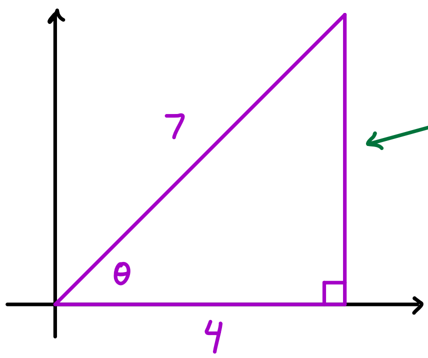
You may need to evaluate expressions involving trig functions and their inverses. We can do this using triangles and trig identities.

Ex: Evaluate the following.

(a) $\sin\left(\cos^{-1}(4/7)\right)$

Solution: Want to know $\sin\theta$ where $\theta = \cos^{-1}(4/7)$.

(so $\cos\theta = 4/7$)



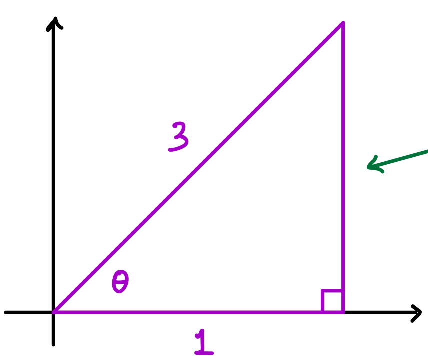
$\sqrt{7^2-4^2} = \sqrt{33}$

$\therefore \sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \boxed{\dfrac{\sqrt{33}}{7}}$

(b) $\sin\left(2\cos^{-1}(1/3)\right)$

Solution: Want to know $\sin 2\theta$ where $\theta = \cos^{-1}(1/3)$.

(so $\cos\theta = 1/3$)



$\sqrt{3^2-1^2} = \sqrt{8}$

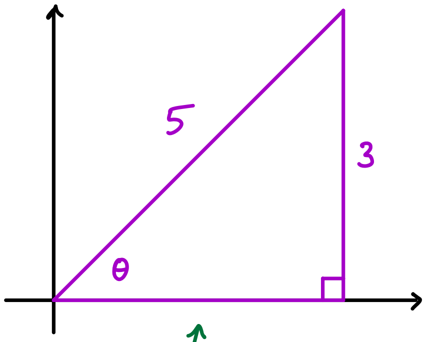
$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\dfrac{\text{opp}}{\text{hyp}}\right)\left(\dfrac{\text{adj}}{\text{hyp}}\right) = \boxed{\dfrac{2\sqrt{8}}{9}}$

(c) $\tan\left(2\sin^{-1}(3/5)\right)$

<u>Solution:</u> Want to know $\tan(2\theta)$ where $\theta = \sin^{-1}(3/5)$

($\text{so } \sin\theta = 3/5$)



$\sqrt{5^2-3^2} = 4$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$$

$$= \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2(3/5)(4/5)}{(4/5)^2 - (3/5)^2}$$

$$= \frac{24/25}{7/25} = \boxed{\frac{24}{7}}$$