

# **TEACHING DOSSIER**

Prepared By  
Zachary Cramer

PhD Candidate  
Department of Pure Mathematics  
University of Waterloo

March, 2019

# Table of Contents

1   Teaching Philosophy Statement .....	1
2   Teaching Experience .....	2
2.1. Courses Taught .....	2
2.2. Teaching Assistantships.....	3
2.3. Math Circles.....	4
2.4. Teaching Techniques for Mathematicians .....	4
2.5. Faculty of Mathematics Teaching Seminar.....	4
3   Teaching Strategies.....	4
3.1. In the Classroom .....	4
3.2. Assessment Methods.....	5
4   Evaluation of Teaching.....	6
4.1. Evaluation from Students.....	6
4.2. Evaluation from Faculty.....	8
5   Professional Development.....	9
5.1. Fundamentals of University Teaching .....	9
5.2. Certificate in University Teaching .....	9
6   Future Goals .....	9
 <b>Appendicies</b>	
A. Sample Course Outline .....	11
B. Sample Midterm Exam.....	13
C. Teaching Assistant Award.....	16
D. Teaching Observation Report.....	17
E. Sample <i>Kahoot!</i> Questions .....	19

# 1 | Teaching Philosophy Statement

You would be met with a nasally laugh if you told me at the age of fifteen that I would one day pursue a career in mathematics. For years I viewed the subject as intimidating, dry, and unmotivated. I struggled to develop intuition for abstract concepts, and often found myself asking every math instructor's favourite question: "When am I ever going to *use* this?" My attitude began to change toward the end of high school; passionate, dedicated teachers helped me to build interest and intuition through engagement and motivation. They challenged me to appreciate not only the usefulness of mathematics, but also its intrinsic beauty.

Today, as an educator and mathematician myself, I know that the difficulties I experienced as a student are far from unique. It is therefore my responsibility to engage my learners by motivating abstract concepts, and providing the means to apply these concepts within their own disciplines. I accomplish these goals by connecting new information to students' existing knowledge, developing intuition and understanding through the use of active learning, and creating a learning environment that encourages intellectual risk-taking and mathematical experimentation.

Students develop a deeper understanding of core concepts when new ideas build upon their existing knowledge. Therefore, I structure my teaching to establish connections within the course material. I begin each class with a brief review of the previous lesson, and end with a question or observation that motivates concepts to come. Furthermore, I use creative assignment problems to link related content. For instance, I introduced matrix inverses in my linear algebra course by building on students' prior understanding of systems of linear equations. I challenged my students to use a single augmented matrix to express each of the standard basis vectors in  $\mathbb{R}^3$  as a linear combinations of three given vectors. Unbeknownst to them, their calculations were in fact determining an inverse matrix. When discussing inverses much later in the course, my students excitedly realized that they had worked with similar ideas in the past. Since adopting this strategy, I notice that students ask more meaningful questions and exhibit stronger intuition overall.

Students learn best when they actively engage in the learning process. For this reason, I structure my lessons around learning activities that encourage participation and collaboration. I use interactive class discussions to develop new concepts and foster critical thinking in my learners. Prompts like "Where have we seen this before?" or "What could go wrong this case?" help to direct these discussions toward our desired learning objectives. In addition, I provide students with opportunities to apply their understanding through collaborative problem solving activities. I use game-based platforms such as *Kahoot!*—an educational trivia app for smartphones—to offer a method of participation that is comfortable for more introverted learners. Students have stated in mid-term surveys and course evaluations that this hands-on approach to learning helps to hold their attention and reinforce their understanding.

I believe that students learn mathematics by doing mathematics. Furthermore, I know that students are more willing to apply new techniques and non-standard arguments on low-stakes assessments such as assignments or quizzes, than on more sizable evaluations such as exams. Therefore, I structure my courses around frequent, low-weight assessments to encourage experimentation with creative approaches to problem solving. This structure fosters a learning environment where students are willing to take risks and learn from their mistakes. In addition, it enables me to closely monitor my students' progress and offer guidance throughout the term. The feedback my students receive from these low-risk assessments helps to raise awareness for alternative solutions and dangerous pitfalls. Consequently, they are less likely to make common mistakes on midterm and final exams.

As a student, I discovered beauty in mathematics through the dedication and enthusiasm of my teachers. Seeing my students develop this same joy in learning is my greatest reward as an educator.

## 2 | Teaching Experience

### 2.1. Courses Taught

#### *Math 106 – Applied Linear Algebra (University of Waterloo)*

I have coordinated and taught *Math 106*, a first course in linear algebra for students outside of the Mathematics, Science, and Engineering faculties. My students were primarily first year undergraduates in Economics or Geomatics. The course covered the basics of vector geometry, systems of linear equations, linear transformations, determinants, eigenvalues and eigenvectors, and applications. The course outline is included in **Appendix A**.

Each week the students attended three 50-minute lectures, as well as a 50-minute tutorial led by the graduate teaching assistant for the course. The tutorials provided opportunities for students to collaborate with their peers to solve problems relating to the lecture content. Each tutorial concluded with a 1-question quiz designed to assess the students' individual understanding of this material. In addition to the tutorials, students were provided with weekly assignments to solidify their understanding of the deeper ideas from class. Our Friday classes concluded with a 10-minute online *Kahoot!* game as a review of the week's material. Sample *Kahoot!* questions are included in **Appendix E**.

As the instructor and coordinator of the course, I designed and delivered lectures; structured tutorials; created weekly assignments, quizzes, and *Kahoot!* games; and prepared two midterms and a final exam. A sample midterm exam is provided in **Appendix B**. Beyond developing assessments and lecture materials, I provided additional assistance to my students by email and in weekly office hours. I also coordinated two teaching assistants throughout the term.

*Date:* Fall 2018

*Audience:* 67 undergraduate students

#### *Math 124 – Calculus and Vector Algebra for Kinesiology (University of Waterloo)*

In the fall of 2017 I taught *Math 124*, an introductory calculus and vectors course required for undergraduates in the Faculty of Applied Health Studies. The first half of the course was devoted to an extensive review of high school pre-calculus and a treatment of the basic concepts from differential calculus: limits, continuity, and derivatives and their applications. The second half of the course provided students with an introduction to integral calculus and the fundamentals of vector geometry in 2 and 3 dimensions.

Each week the students attended three 50-minute lectures, as well as a 50-minute tutorial led by the course teaching assistants. In the tutorial, students completed a short assignment designed to reinforce their knowledge of the previous week's material.

The course was coordinated by another instructor in the Faculty of Mathematics. As the instructor in charge of my section, I prepared and delivered lectures, provided assistance to my students in office hours and over email, and oversaw the responsibilities of two graduate teaching assistants. I also created problems for the midterm and final exam, and assisted in reviewing and revising these assessments.

*Date:* Fall 2017

*Audience:* 220 second year undergraduates in Kinesiology

### *Math 138 – Calculus II for Honours Mathematics (University of Waterloo)*

For a period of 5 non-consecutive weeks I taught as an interim instructor for *Math 138*, a second course in calculus offered to students in mathematics and computer science at the University of Waterloo.

In the first two weeks of this appointment, I designed and delivered seven 50-minute lectures on sequences and series for two sections of the course. During this time I also held weekly office hours and assisted in invigilating the course's midterm exam. In the final three weeks, I created and delivered six lectures on Taylor series and developed review material for the final exam. I oversaw the responsibilities of four graduate teaching assistants, provided students with additional help in office hours, and invigilated the course's final exam.

*Date:* Fall 2016 (5 weeks)

*Audience:* 100 undergraduate students in mathematics

## **2.2. Teaching Assistantships**

### **University of Waterloo**

As a graduate student at the University of Waterloo, I am given the opportunity to work as a teaching assistant for courses offered by the Faculty of Mathematics and the Department of Pure Mathematics.

The majority of my teaching assistantships have involved courses offered at the first-year level: *Advanced Calculus II*, *Calculus I for the Sciences*; *Honours Calculus II*; *Linear Algebra for Engineering*; and *Algebra for Honours Mathematics*. Within these roles I have graded midterm and final exams, coordinated teams of undergraduate grades, held office hours in the faculty's drop-in tutorial center, and led tutorials for audiences of between 40 and 150 students.

I have work as a teaching assistant for 4 upper-year courses in the Department of Pure Mathematics: *Euclidean Geometry*; *Complex Analysis*; *Measure and Integration*; and *Lebesgue Measure and Fourier Analysis*. For each course, I assisted in grading assignments, midterms, and final exams. The course in Euclidean geometry also featured an interactive tutorial that I helped to organize and facilitate each week.

I have also worked as a teaching assistant for an online course in calculus I offered through the University of Waterloo's *Master of Mathematics for Teachers* (MMT) program. This is a professional program designed to provide high school math teachers with a deeper understanding of the mathematics underlying their curricula. As a teaching assistant for this course, I graded assignments, managed discussion boards, and offered extensive written feedback to the participants.

### **University of Windsor**

I worked as teaching assistant for a period of two years while completing my undergraduate degree at the University of Windsor. I began this role as a grader and grading coordinator for an introductory course in differential calculus. My success resulted in opportunities to lead weekly tutorials for upper-year courses in real analysis, complex analysis, and Fourier analysis. These tutorials were attended by 20-50 students. I created problem sets for each tutorial, and provided additional assistance in the drop-in tutorial center.

## **2.3. Math Circles**

I have developed three mini-courses for the University of Waterloo's Math Circles program, a free weekly enrichment activity for students in grades 6-12, organized by the Centre for Education in Mathematics and Computing (CEMC). For each course, I designed and delivered 2-3 workshops to an audience of 60-70 students in grades 9/10. Each workshop consisted of a 1 hour lecture, interspersed with 1 hour of group problem solving. The lectures were recorded by the CEMC, and are freely available on their Math Circles website. The lecture notes, problem sets, and solutions sets that I developed are also freely available.

My first course was offered in the winter of 2017, and covered selected topics from Graph Theory. The second course I designed was offered in the winter of 2018, and featured two 2-hour workshops on linear Diophantine equations. Most recently, I designed two workshops on sequences and series, which I presented in the winter of 2019.

## **2.4. Teaching Techniques for Mathematicians**

Together with members of the University of Waterloo's Faculty of Mathematics, I developed a 12-week instructional seminar on university teaching aimed at new graduate student instructor. The first iteration of the seminar occurred weekly in the winter of 2018 and included roughly 15 participants. By popular demand, a second iteration is currently being offered and now includes over 25 participants.

Our workshops address various topics in undergraduate mathematics education, including lesson design, assessment methods, active learning strategies, and course administration. Participants also completed three microteaching sessions, as well as a guest lecture for a course similar to one they may teach in the near future. As an organizer, I have the opportunity to observe these lectures and provide detailed verbal and written feedback to the participants.

## **2.5. Faculty of Mathematics Teaching Seminar**

In June of 2018, I presented at a teaching seminar organized by members of the University of Waterloo's Faculty of Mathematics. The seminar met bi-weekly and featured various topics related to teaching mathematics at the undergraduate level. Each meeting consisted of a 30-minute presentation followed by a 20-minute discussion. I spoke on active learning and flipped classrooms in introductory math, a topic that I researched as part of the CUT program at the University of Waterloo's Centre for Teaching Excellence (see section 5.2).

# **3 | Teaching Strategies**

## **3.1. In the Classroom**

As an instructor, it is my goal to provide an engaging and thorough treatment of course content, while ensuring that my students leave each and every lesson with the knowledge and confidence to apply the subject matter on their own. Therefore, I use a variety of teaching methods to actively direct my students toward a set of well-defined learning objectives.

In order to connect the material from each lesson with my students' prior knowledge, I begin every lecture with a 5-minute review of content from the preceding class. This quick recollection process helps to fit each lesson within the course as a whole. I direct the class by providing my students with 2-3 concrete learning objectives that will be met by the end of the lesson. The learning objectives remain visible during the entire class period, and are marked with a check as they are completed. This strategy allows students to keep track of what has been covered, what is currently being discussed, and what is yet to be addressed.

Students learn best when they actively engage in the learning process. For this reason, I limit my expositions of new content to 15-20 minute mini-lessons separated by opportunities for active learning. During the mini-lessons, I lead my learners to new ideas or applications through a directed dialogue. Using prompts like "Does this look expression familiar?" or "What theorem can help us here?" encourages participation and helps to create a sense of independent discovery in my learners. In the learning activity that follows, students work either individually or in small groups to solve related problems. I tour the room to connect with my students one-on-one, and offer guidance to groups in need. I conclude the activity by summarizing the solution with the class.

Whenever possible, I solidify new information through simply demonstrations or real world applications. One of my favourite examples of this approach was used when I taught center of mass as an application of integration in first-year calculus. The students and I first found the center of mass for the region enclosed by the functions  $y=x^2$  and  $y=x^{1/2}$ . I then revealed a cardboard cut-out of this region, and balanced it on the tip of a pen positioned at the computed point. As a second example, I used interactive graphing software when teaching linear algebra to provide a visualization of various linear transformations in 2- and 3-dimensions. This fun, hands-on interface helped my students to develop a better geometric understanding of matrices and linear maps. This software was also used in describing geometric interpretations for other notions throughout the course, including eigenvalues, eigenvectors, and determinants.

### 3.2. Assessment Methods

When designing my courses, I incorporate a variety of assessment methods to promote mathematical thinking, connect various ideas throughout course, and evaluate students' abilities at multiple levels of cognitive complexity.

I include formative comprehension checks to assess my students' knowledge and understanding of core concepts. For instance, I end my tutorials with a 10-minute quiz to ensure that my students have a good working knowledge of the relevant definitions and examples. While the quizzes do contribute to a small percentage of the final grades, they are primarily used as a means for me to monitor my students' progress and offer regular feedback. In a similar vein, I conclude my final lesson of each week with a game of *Kahoot!* so that I may gauge my students' understanding of the week's material.

Building mathematical intuition requires both practice and time. For this reason, I implement weekly or bi-weekly assignments that provide an opportunity for my students to attempt complex problems and train their mathematical reasoning skills. I include questions that require students to apply mathematical theory, synthesize ideas, and evaluate the validity of conjectures and sample arguments. As a result, I am able to facilitate higher levels of learning through my assignments.

I believe that when carefully designed, assignments can be valuable teaching aids as well as platforms for evaluation. Specifically, I have found that by building on existing ideas through suggestive assignment problems, these assessments can help to connect ideas within the course and provide insight into future material. For example, when assigning problems in first-year linear algebra that apply students' knowledge

of systems of linear equations, I instructed my learners to express each of the standard basis vectors in  $\mathbb{R}^3$  as a linear combination of three given vectors. To reduce the computations, I advised the students to work with a single augmented matrix containing all three standard basis vectors on the right-hand side. Unbeknownst to them, their calculations were in fact determining an inverse matrix! When this notion was introduced much later in the course, the students excitedly realized that they have worked with similar ideas in the past. This “Aha!” moment adds familiarity to an otherwise unfamiliar topic, and fosters a deeper understanding of the material overall.

## 4 | Evaluation of Teaching

### 4.1. Evaluation from Students

The following tables provide a summaries of my instructor evaluations from the University of Waterloo. Each category is rated on a scale of 1 to 5, with 1 being the worst and 5 the best. A score in **red** indicates the lowest of the category, while a score in **blue** indicates the highest.

<i>Semester</i>	Fall 2017	Fall 2018
<i>Course</i>	<i>Math 124 Calculus and Vector Algebra for Kinesiology</i>	<i>Math 106 Applied Linear Algebra I (Coordinator)</i>
<i>Student Enrolled</i>	220	67
<i>Percent Responding</i>	62.7%	74.6%
<i>Tompa Score</i>	4.6	4.8
<i>Organization</i>	4.78	4.90
<i>Explanations</i>	4.78	4.82
<i>Ability to Answer Questions</i>	4.73	4.88
<i>Visual Presentation</i>	4.68	4.84
<i>Oral Presentation</i>	4.82	4.90
<i>Availability</i>	4.77	4.86
<i>Effectiveness</i>	4.77	4.88

Overall, my style of instruction appears to have been quite effective for these audiences. My students deviated significantly in their prior exposure to course content, so I always attempted to find a balance between elementary, down-to-earth explanations for my less experienced learners, while still offering my



more advanced learners a fresh perspective on familiar topics. Thus, I was particularly pleased to see my high scores in the *Explanation*, *Visual Presentation*, and *Oral Presentation* categories.

My learners strongly favoured my passion for mathematics and enthusiasm in the classroom. Many students indicated that these aspects of my teaching helped to promote interest in the subject matter and hold their attention. As one student explained, *“He was able to remain enthusiastic during lectures which allowed me to remain attentive even through tough concepts.”*

Another aspect of my teaching that students found especially effective was my interactive, problem-based style of instruction. By allocating time for student input and in-class problem solving, I was able to build rapport with my learners, and correct misconceptions immediately as they arose. One student commented on this approach by saying that

*“[Zack] gave us plenty of in-class practice time, in which he walked around providing help or talked to the students to connect with them. He is also very approachable and took the time to develop that comfortability with his students.”*

In order to more effectively provide this level of engagement with large audiences, I have integrated technology-based games such as *Kahoot!* into my classroom routine. My students have expressed a strong preference for this method of instruction, and I look forward to implementing it in future courses.

Finally, it is interesting to note that my scores in all categories increased between my two courses. In part, this is likely due to the fact that my learners in *Math 106* were primarily from math focused disciplines such as economics and geomatics, while the students in *Math 124* were from Kinesiology. I do believe, however, that this increase also reflects my growth as an instructor and the refinement of my teaching practices following *Math 124*.

### **Additional Selected Responses:**

- *“As a visual learner, I appreciate that Zack took the time to write everything out neatly using different colours and used diagrams. He even brought in props to demonstrate certain concepts.”*
- *“He was very respectful and courteous. He even encouraged the class to ask questions. . . . He created a very positive and comfortable environment that made students feel safe participating.”*
- *“It is obvious that his best interest is for the students and always wants to see them succeed.”*
- *“I have always hated math growing up and I dreaded the thought of taking this course. However, after taking this course I may have changed my perspective. . . . Zack made this course fun and his passion for the subject really showed.”*
- *“Not many professors continue to structure and organize the course to benefit the learning of their students. This instructor did an amazing job of being organized and coherent throughout.”*
- *“Every lecture started with a recalling of what we went over in the last class making lectures flow from one day to another.”*

## 4.2. Evaluation from Faculty

### Instructor Evaluations

Through the Certificate of University Teaching program at the University of Waterloo's Centre for Teaching Excellence (see section 5.2), I received two teaching observation reports from members of the Faculty of Mathematics. The first observation was conducted by Dr. Brian Forrest of the Department of Pure Mathematics, and his complete report is included in **Appendix D**. The second observation was conducted by Mr. Ian VanderBurgh, Director of the Center for Education in Mathematics and Computing (CEMC).

Both observers remarked that I presented the content very clearly and at an appropriate pace for my audience. They found that my decisions to move around the room and allocate time for my students to solve problems collaboratively were effective in maintaining student engagement.

One area of improvement identified by Dr. Forrest concerned my lecture preparation routine. Prior to his observation, my preparation included practicing each lecture twice before delivering it in my classroom. While this approach makes for a very polished exposition of the content, Dr. Forrest commented that it is very time consuming, and may lead add too much rigidity to the lesson. Following Dr. Forrest's recommendations, I have successfully reduced my per-lecture preparation time by practicing only the technical aspects of each lesson. As a result, I am now better able to adjust the pace and direction of my lessons when interesting questions or suggestions are raised by my students.

### Teaching Assistant Evaluations

Since 2018, graduate teaching assistants in the Department of Pure Mathematics at the University of Waterloo have received written performance assessments from their course coordinators. My evaluations are summarized below.

Each category is assigned a rating of *Excellent*, *Very Good*, *Good*, *Satisfactory*, or *Unsatisfactory*.

<i>Semester</i>	Winter 2018	Spring 2018
<i>Course</i>	<i>PMath 352 Complex Analysis</i>	<i>PMath 320 Euclidean Geometry</i>
<i>Overall</i>	Excellent	Excellent
<i>Quality of Work</i>	Excellent	Excellent
<i>Timeliness</i>	Excellent	Excellent
<i>Communication</i>	Excellent	Excellent

The *Euclidean Geometry* course described above was developed and taught by Dr. Benoit Charbonneau of the Department of Pure Mathematics, and was offered for the first time in the summer of 2018. Based on my reputation as a reliable and effective teaching assistant, Dr. Charbonneau personally requested that I assist with the first iteration of the course. He included the following written feedback on my performance:

*“Zack, your help in delivering the first instance of PMath 320 was exceptional. You have been a very positive asset and an encourager. I know I could seek you for honest and well thought feedback on any aspects of the course. You particularly shone in the interactive tutorials in your interactions with the students. Thank you for being part of the team.”*

In recognition of my work with Dr. Charbonneau and my overall record of dedication and excellence as a teaching assistant, the Department of Pure Mathematics presented me with the *2018 Outstanding Teaching Assistant Award*. A copy of the award is included in **Appendix C**.

## 5 | Professional Development

### 5.1. Fundamentals of University Teaching

I have successfully completed the *Fundamentals of University Teaching* (FUT) program through the Center for Teaching Excellence (CTE) at the University of Waterloo. As a part of this program, I attended six workshops focusing on developing classroom management and delivery skills, creating effective lesson plans, identifying and practicing active learning strategies, and increasing one’s ability to give and receive effective feedback. Moreover, I participated in three microteaching sessions, each consisting of a short interactive lesson followed by peer feedback.

The FUT program has helped me to improve my teaching skills both inside and outside the classroom. In particular, I learned to direct my lessons through the use of explicit learning objectives. Additionally, I gained valuable experience in promoting excitement and engagement in my learners through the use of active learning strategies.

### 5.2. Certificate in University Teaching

I have completing the *Certificate of University Teaching* (CUT) program at the University of Waterloo’s CTE. The program consists of three courses, each focusing on a particular aspect of post-secondary education. The first course includes four workshops on student learning, interactive teaching, assessment methods, and course design. The second features an expository research project on an aspect of teaching and learning in higher education, as well as the completion of a teaching dossier. The final course consists of two teaching observations by members of the participant’s faculty, or by members of the CTE. My observation report by Professor Brian Forrest of the Department of Pure Mathematics is included in **Appendix D**.

## 6 | Future Goals

I am very excited to take part in future teaching opportunities within the field of mathematics. In addition to refining my abilities as an instructor for introductory courses in calculus and algebra, I soon hope to gain further experience in course coordination and undergraduate curriculum development. Since building personal connections with students is among the most rewarding aspects of my service activities, I look forward to further developing my skills as an academic advisor and student mentor.

Throughout my teaching career, I will continue to experiment with instructional methods that promote participation and collaboration in my learners. Specifically, I would like to maximize the impact of laptops and smartphones on student learning by exploring a variety of technology based learning education platforms. In order to maintain awareness of new and effective teaching techniques, I will continue to participate in workshops and conferences devoted to teaching and learning in higher education.

## A. Sample Course Outline

### MATH 106 - Applied Linear Algebra I Fall 2018

Instructor: Zack Cramer  
 Email: zcramer@uwaterloo.ca  
 Office: MC 5422

Office Hours: Wednesday 9:00-10:00  
 Thursday 10:30-12:00

**Lectures:** MWF, 3:30-4:20 in E2 1736.

**Prerequisites:** MATH 103 or 4U Calculus and Vectors.

**Syllabus:** Vectors space geometry, including lines, planes, projections, and cross products. Subspaces of  $\mathbb{R}^n$ . Systems of linear equations. Matrix algebra. Linear transformations, matrix mappings, the rank-nullity theorem. Determinants. Eigenvalues and diagonalization. Applications.

**Textbook:** The following textbooks are on course reserve at the Davis Centre library, and the first is available for purchase at the UW bookstore. It is not required that you purchase either text.

Primary reference: *Introduction to Linear Algebra for Science and Engineering, 2nd ed.*  
 Daniel Norman and Dan Wolczuk

Secondary reference: *Introduction to Linear Algebra, 5th ed.*  
 Gilbert Strang

**Course Website:** All relevant information can be found on our LEARN webpage (<https://learn.uwaterloo.ca>).

**Grading:** Grades will be computed according the the following scheme.

Assignments: 15%	Midterm Exam 1: 20%	Final Exam 40%
Tutorial Quizzes: 5%	Midterm Exam 2: 20%	

**Assignments:** There will be a total of 10+1 weekly assignments, which will be submitted electronically via Crowdmark (<https://crowdmark.com>). Assignments are due at **12:00 noon** on the dates below. Late assignments will NOT be accepted. The lowest assignment score will be dropped.

Assignment	Due date	Assignment 5	October 19
Assignment 0	September 11	Assignment 6	October 26
Assignment 1	September 14	Assignment 7	November 2
Assignment 2	September 21	Assignment 8	November 9
Assignment 3	September 28	Assignment 9	November 23
Assignment 4	October 5	Assignment 10	November 30

**Tutorials:** Tutorials will take place on Wednesdays from 1:30-2:20 in PHY 235, and will begin on September 12th.

In tutorials, the TA will present problems designed to supplement the content from class. You will be given an opportunity to attempt these problems yourselves. You are encouraged to work with those around you and ask the TA for help if needed. The last 10-15 minutes of each tutorial will consist of a 1-2 question quiz that will be completed individually.

**Midterm Exam 1:** Written in the tutorial on Wednesday, October 17.

**Midterm Exam 2:** Written in the tutorial on Wednesday, November 14.

**Final Exam:** Written on Thursday, December 6 from 12:30-3:00 in MC 1085.

**Missed examination policy.** For any missed examination, you must have a valid reason (for instance, illness) and appropriate supporting documentation (i.e. University of Waterloo Verification of Illness Form). A copy of these forms must be given to your instructor as soon as possible. Absence for the either midterm exam will then result in the weight being shifted to the final exam. There will be no deferred (make-up) midterms. Absence for the final exam may result in a grade of INC at the discretion of your instructor. To be considered for an INC, you must have passing grades on the midterm exams and a strong average over all assignments and quizzes. (See <http://ugradcalendar.uwaterloo.ca/page/Regulations-Accommodations>)

**Academic Integrity:** In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility.

[Check [www.uwaterloo.ca/academicintegrity/](http://www.uwaterloo.ca/academicintegrity/) for more information.]

**Grievance:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, <http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm>. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

**Discipline:** A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean.

For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, <http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm>. For typical penalties check Guidelines for the Assessment of Penalties, <http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm>.

**Appeals:** A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, <http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm>.

**Note for students with disabilities:** AccessAbility Services, located in Needles Hall, Room 1401, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AccessAbility Services at the beginning of each academic term.

## B. Sample Midterm Exam

# CROWDMARK



## UNIVERSITY OF WATERLOO

Please print in pen:  
Waterloo Student ID Number:

--	--	--	--	--	--	--	--

WatAM/Quest Login Userid:

--	--	--	--	--	--	--	--

Times: Wednesday 2018-11-14 at 13:30 to 14:20 (1:30 to 2:20PM)

Duration: 50 minutes

Exam ID: 3974372

Sections: MATH 106 LEC 001

Instructors: Zack Cramer

### Closed Book

Candidates may bring no aids (no calculators).

#### Instructions

1. Please make sure you are sitting at the right exam. Your picture should be at the top of this page!
2. This exam consists of 5 questions worth 35+1 marks total. There are 7 pages including the cover page. Check that you have them all. **DO NOT REMOVE ANY PAGES.**
3. Answer all questions in the spaces provided, using the last page for overflow or rough work. If your answer spills out of the space provided, indicate where it can be found.
4. With the exception of question 1, thoroughly justify all of your answers.
5. This is a closed-book exam. You may write in pen or pencil, but pencil is preferred. Please write clearly and legibly. No calculators are permitted.
6. This exam will feature both Pokémon and cats eating ice cream.

FOR EXAMINER'S USE ONLY	
Question	Mark
1	/4
2	/6
3	/8
4	/8
5	/9
Total	/35+1



[4] 1. **Multiple Choice:** Circle i., ii., iii., or iv. to indicate a correct answer. No justification is required.

[1] (a) If  $A$  is a  $3 \times 5$  matrix,  $B$  is a  $2 \times 5$  matrix, and  $C$  is a  $2 \times 3$  matrix, then  $C(BA^T)^T$  is

- (i)  $2 \times 5$ ;
- (ii)  $2 \times 2$ ;
- (iii)  $5 \times 2$ ;
- (iv) not defined.

[1] (b) If  $A$  is a  $5 \times 3$  matrix with rank 3 then

- (i) the columns of  $A$  span  $\mathbb{R}^5$ ;
- (ii) the columns of  $A$  are linearly dependent;
- (iii) there is a vector  $\vec{b}$  such that  $[A | \vec{b}]$  is inconsistent;
- (iv) the homogeneous system  $[A | \vec{0}]$  has infinitely many solutions.

[1] (c) Which of the following is impossible for a system of 2 equations in 3 variables?

- (i) The solution is a point.
- (ii) The solution is a line.
- (iii) The solution is a plane.
- (iv) There is no solution.

[1] (d) If  $A$  is an  $m \times n$  matrix and  $RREF(A)$  has a row of 0's, then the homogeneous system  $A\vec{x} = \vec{0}$

- (i) has infinitely many solutions;
- (ii) has a unique solution;
- (iii) has no solution;
- (iv) may have a unique solution or infinitely many solutions.

[6] 2. Consider the matrix  $A$  and its  $RREF$  given by

$$A = \begin{bmatrix} 4 & 8 & 1 & 11 & 1 \\ 2 & 4 & 1 & 5 & 0 \\ 3 & 6 & 1 & 8 & 1 \end{bmatrix} \quad \text{and} \quad RREF(A) = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

[1] (a) The rank of  $A$  is \_\_\_\_\_.

[1] (b) The solution to the homogeneous system  $[A | \vec{0}]$  has \_\_\_\_\_ parameters.

[2] (c) Do the columns of  $A$  span  $\mathbb{R}^3$ ? Explain.

[2] (d) Are the columns of  $A$  linearly independent? Explain.



---

[8] 3. Consider the set  $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} \right\}$

[2] (a) Is  $\mathcal{B}$  a basis for  $\mathbb{R}^4$ ? Explain.

[4] (b) Write  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$  as a linear combination of the vectors in  $\mathcal{B}$ , or show that this isn't possible.

[2] (c) Suppose that  $L : \mathbb{R}^4 \rightarrow \mathbb{R}^2$  is a linear mapping such that

$$L(1, 2, 3, 1) = (1, 0), \quad L(2, 4, 7, 2) = (2, 1), \quad \text{and} \quad L(1, 3, 3, 1) = (0, 1) .$$

Determine  $L(1, 0, 2, 1)$ .

[8] 4. Consider the mapping  $L(x_1, x_2) = (2x_1 - x_2, 0, 3x_1)$ .

[2] (a) The domain of  $L$  is \_\_\_\_\_ and the codomain of  $L$  is \_\_\_\_\_ .

[3] (a) Show that  $L$  is linear.

[2] (c) Determine the matrix  $[L]$ .

[1] (d) Use  $[L]$  to compute  $L(1, 4)$ .

[9] 5. Consider the line  $L$  given by  $\vec{x} = t \begin{bmatrix} -1 \\ 2 \end{bmatrix}, t \in \mathbb{R}$ .

[1] (a) A normal vector for  $L$  is  $\vec{n} =$  \_\_\_\_\_ .

[3] (b) Determine the matrix for the linear map on  $\mathbb{R}^2$  that reflects vectors over  $L$ .

[3] (c) The vector  $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$  is reflected over  $L$  and then rotated counterclockwise by  $\pi/2$ . What is the resulting vector?

[2] (d) Find a non-zero  $2 \times 2$  matrix  $A$  such that for every  $\vec{x} \in \mathbb{R}^2$ ,  $A^2\vec{x}$  is orthogonal to  $\vec{x}$ .

[1] **Bonus:** Draw a your favourite Pokémon. Or a cat eating ice cream. Or both.

[Don't spend more than a minute on this unless you've finished the rest of the exam]

C. Teaching Assistant Award



*Outstanding Teaching Assistant Award*

AWARDED TO

**Zachary Cramer**

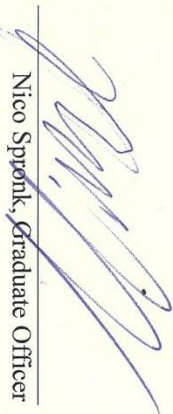
In honour of your outstanding performance and dedication  
in the performance of your teaching assistantship duties.

September 20, 2018

  
David McKinnon, Chair



Department of Pure Mathematics  
University of Waterloo  
Waterloo, Ontario, Canada

  
Nico Spronk, Graduate Officer

## D. Teaching Observation Report

Private and Confidential

### Observation Report 1 Zachary Cramer

---

**Event Observed:** MATH 124 Lecture

**Date Observed:** Monday, October 16<sup>th</sup>, 2017

**Location:** AHS 1689

**Time:** 03:30-04:20

**Number of Students Present:** Approximately 160-170

**Observer:** Brian Forrest, Pure Mathematics, Professor

---

#### Plan for Teaching Event:

By the end of this lesson, students will be able to state the Chain Rule; identify settings in which the Chain Rule can be used; and apply the Chain Rule to find the derivative of a composition of differentiable functions.

---

#### Aspects to Maintain:

1. Your content delivery was excellent. Your voice was clear and you spoke at a manageable pace. The notes you presented were excellent. It can be a challenge for a mathematician to deliver content via a document camera since this is not our usual mode of delivery, but you managed this extremely well. The pace of your content delivery was also very good for the course level. This is a real key for success. Too slow and the students will drift. Too fast and they will simply go into note-taking mode and miss what you have to say. Overall you did a very good job in a rather challenging classroom.
2. Your choice to give the students a few minutes to work on problems worked very well in this course. I was particularly impressed by how thoroughly you covered the large room. It also seemed to me that students were working on the problems you posed. This course has a fair number of basic questions that are well suited to in-class practice time as a means of reinforcing the content. It also seemed that the students were very at ease with you when you walked amongst them. In addition, these questions gave you a chance to move throughout the room rather than being a captive of the document camera.
3. The students were taking advantage of your pre-posted notes to help follow along with the lecture. This can both work for you or against you depending on how much additional information you give them in your lecture. In your case, I thought you added just the right amount of additional information to keep them attentive but to not overwhelm them.

**This confidential report is based on an observation of a single teaching event, and is intended for the personal use of Zachary Cramer in support of his teaching activities.**

**Private and Confidential****Targets for Change and Methods for Improvement:**

1. Your lecture preparation time needs to be managed differently. To find balance between teaching and your other duties you should be able to cut down your per-lecture practice time significantly. You should try making a bullet point summary of the proposed lecture and using that as your key preparation tool. This is also a question of confidence. You have established a strong rapport with your class it would appear. They know that you are knowledgeable and well prepared so they can forgive minor imperfections. Over time you should also try and be comfortable going off script. When a student asks a particularly insightful question it can be a real learning opportunity to take a detour from your planned lecture and address the question in a deeper way than you may normally have time to do. If your lecture is too carefully planned it can be a challenge to deviate from the plan.

2. You should always be conscious of the acoustics in the room with regards to student questions. While you did a good job in this respect you should always remember to repeat the questions to the whole class prior to answering them. The students were engaged but did not ask a lot of questions. It is always a good idea to think of additional ways to get them involved in your lectures.

3. Based on this lecture alone there are not a lot of improvements I can suggest in terms of the technical aspects of your delivery. But one can always put more time into the big picture preparation. How does one individual lecture tie into other lectures either in the past or to be delivered in the future? In choosing approaches or illustrative examples think about where you have been and where you are going in the class.

**Additional Comments:** Over all this was an excellent lecture. It was polished in terms of delivery but more importantly you did a very good job of explaining the key ideas needed for the students to understand how the Chain Rule works.

**Response Paper (2-3 pages, single-spaced)**

**This confidential report is based on an observation of a single teaching event, and is intended for the personal use of Zachary Cramer in support of his teaching activities.**

## E. Sample Kahoot! Questions

Which of the following is a subspace?

51

(A)

(B)

0 Answers

A only
  B only
  Both A and B
  Neither

Which of the following is equal to  $\text{Proj}_{\mathbf{x}}\mathbf{y}$ ?

50

Kahoot!

0 Answers

$\text{Proj}_{2\mathbf{x}}\mathbf{y}$ 
  $\text{Proj}_{\mathbf{x}}(2\mathbf{y})$ 
  $\text{Proj}_{\mathbf{x}}(-\mathbf{y})$ 
  $\text{Proj}_{\mathbf{y}}\mathbf{x}$

Which is impossible for a system of 2 equations in 3 variables? **HINT:** think geometrically!

44

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

0 Answers

The solution is a plane.
  The solution is a line.
  The solution is a point.
  There is no solution.



Let  $S$  be the map that reflects vectors over the  $x_2$ -axis. Select the correct hedgehog.

57

$(S \circ R_{\frac{\pi}{2}} \circ S)(\text{hedgehog}) =$

A B C D

0 Answers

Skip

▲ A

◆ B

● C

■ D

Which of the following is FALSE?

3

Kahoot!

0 Answers

Skip

▲ Every matrix mapping is linear.

◆ The line  $y = x - 2$  is a linear mapping from  $\mathbb{R}$  to  $\mathbb{R}$ .

● If  $L$  is a linear mapping, then  $L(0)=0$ .

■ Every linear mapping can be represented by a matrix.

A linear map  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  reflects every vector in the  $x_2$ -axis. Which of the following is  $[L]$ ?

57

A  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  B  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

C  $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$  D  $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

0 Answers

Skip

▲ A

◆ B

● C

■ D