Bernoulli Trial Problems for 2020

1: (2 minutes) If A is a proper subset of B and C is a proper subset of D, then $A \cup C$ is a proper subset of $B \cup D$.

2: (3 minutes) For all positive integers n, $\sqrt[n]{3} + \sqrt[n]{7} > \sqrt[n]{4} + \sqrt[n]{5}$.

3: (3 minutes) There is an infinite set of positive integers such that no matter how we choose some elements of this set, their sum is not a perfect power.

4: (4 minutes) The Fourier transform of an integer $n \geq 9$ is defined to be the positive number b such that the base-b expansion of n has the most number of 4's among all expansions of n. If there is a tie among the bases with the most number of 4's, the average of these bases is the Fourier transform of n.

5: (3 minutes) There is a nine-digit perfect square, in which every digit except 0 appears and whose last digit is 5.

6: (3 minutes) There exists a set S of points in $\mathbb{R}^3$ such that for any plane P in $\mathbb{R}^3$, $P \cap S$ is finite and nonempty.

7: (3 minutes) The number 1280000401 is a prime.

8: (3 minutes) There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x$ has 2019 solutions while $f(f(x)) = x$ has 9102 solutions.

9: (3 minutes) It is possible to load a pair of dice (so they each have customized probabilities of rolling $1, 2, \ldots, 6$) so that the sum takes the values $2, 3, \ldots, 12$ equally likely.

10: (4 minutes) If n is a positive integer, then $\lfloor \sqrt{n} + \sqrt{n+1} \rfloor = \lfloor \sqrt{4n+2} \rfloor$.

11: (4 minutes) There is a regular n-gon, where $n \neq 4$, all of those vertices are lattice points (i.e. the x- and y-coordinates of all the vertices are integers).

12: (4 minutes) $\log_2 3 + \log_3 4 + \log_4 5 > 4$.

13: (4 minutes) A double-sequence of length n where n is a positive even number is a sequence in which $1, 2, 3 \ldots, n/2$ each appears twice, with the second occurrence of r being r positions after its first occurrence. For example, $4, 2, 3, 2, 4, 3, 1, 1$ is a double-sequence of length 8.

14: (5 minutes) Suppose $f : [0, 1] \to \mathbb{R}$ is a function with continuous second derivative with $f(0) =$

$f(1) = 0$ and that $f(x) > 0$ for all $x \in (0, 1)$. Then

$$\int_0^1 \left| \frac{f''(x)}{f(x)} \right| dx > 4.$$

15: (5 minutes) Every closed convex region in the plane of area at least π contains two points whose distance from each other is at least 2.