Bernoulli Trials Problems for 2015

1: The number of positive integers whose digits occur in strictly decreasing order is $2(2^9-1)$.

2: Let n be the smallest positive integer such that $7^n \equiv 1 \bmod 2015$. Then $n \geq 100$.

3: The number $\sqrt[3]{7+5\sqrt{2}}+\sqrt{11-6\sqrt{2}}$ is rational.

4: For every field F and every square matrix A with entries in F, $\text{Row}(A)\cap\text{Null}(A)=\{0\}$.

5: For each $n\in\mathbf{Z}^+$, let x_n be the number of matrices $A\in M_{3\times n}(\mathbf{Z}_3)$ with no two horizontally or vertically adjacent entries equal. Then there exists $n\in\mathbf{Z}^+$ such that x_n is a square.

6: $\prod_{k=1}^{50}\frac{2k}{2k-1}>12$.

7: $\displaystyle\int_0^{\pi/2}\sqrt{2\tan x}\,dx\ >\pi$.

8: A light at position $(0,0,4)$ shines down on the sphere of radius 1 centered at $(3,0,2)$ casting a shadow on the xy-plane. The area of the shadow is greater than 33.

9: There exists a continuous function $f:[0,1]\to[0,1]$ such that for every $y\in[0,1]$ the number of $x\in[0,1]$ for which $f(x)=y$ is finite and even.

10: There exists a polynomial $f\in\mathbf{Q}[x,y]$ such that the map $f:\mathbf{N}\times\mathbf{N}\to\mathbf{N}$ is bijective.

11: There exists a bijective map $f:\mathbf{Z}^+\to[0,1]\cap\mathbf{Q}$ such that $\displaystyle\sum_{n=1}^{\infty}\frac{f(n)}{n}$ converges.

12: For every sequence of real numbers $\{a_n\}$, if $\sum_{n=1}^{\infty}a_n$ converges then so does the series

$$a_1,\ a_2,\ a_4,a_3,\ a_8,a_7,a_6,a_5,\ a_{16},a_{15},\cdots,a_9,\ a_{32},a_{31},\cdots,a_{17},a_{64},\cdots$$

13: Initially, $n=2$. Two players, A and B, take turns with A going first. At each turn, the player whose turn it is can either replace n by $n+1$ or by $2n$. The first player to replace n by a number larger than 130 loses. In this game, player A has a winning strategy.