

The Nearest Point in a Compact Set

6122 [1976, 817]. *Proposed by Albert A. Mullin, Redstone Arsenal, Alabama*

Does there exist a compact set $S \subset E^2$ such that for each $x \in E^2 \setminus S$ there exist precisely two nearest points of S ? Clearly S cannot be convex.

Solution by William J. Gilbert, University of Waterloo, Canada. The answer is no. If S is any compact set in E^2 , it cannot be the whole of E^2 , so there exists a point $y \in E^2 \setminus S$. Let s be a point of S nearest to y . The circle, center y , radius ys , contains no points of S in its interior; since S is compact, its radius is nonzero. Consider any point x lying on the radius between y and s . The circle center x and radius xs contains only one point of S , namely s , and no points of S in its interior. Hence $x \in E^2 \setminus S$ and has only one nearest point of S .

Also solved by Marek Anczura, D. H. Armitage (Northern Ireland), Dennis Berkey, J. M. Borwein (Canada), Dietrich Braess (F. R. Germany), William Bynum & David Stanford, John Cantwell & Raymond Freese & Dana Kamerud, Kenneth Falconer (England), Thomas Gard, Jerrold Griggs, Gustaf Gripenberg (Finland), Donald Hayman, Edward Howorka, Eli Isaacson, Elgin Johnston, Lee Keener (Canada), E. H. Kronheimer (England), O. P. Lossers (Netherlands), Alvin Martin, Mark Meyerson, Lee Mohler, John Morgan II, James Munkres, Edward Ordman, Bruce Peterson, John Rainwater, Adam Riese, Ira Rosenholtz, Martin Schechter, Wolfe Snow,