The Waterloo Region Math Circles
## Fractals Test

9 March 2005   7:30 − 8:30 p.m.

Question 1. The Koch curve is constructed by starting with a line of unit length (at the zeroth stage) and repeatedly replacing each interval

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(a) Find the length at the nth stage of the construction.

(b) What is the fractal dimension of the Koch curve?

(c) Why is this a fractal?

Question 2. The following figures show the first four stages in the construction of a fractal, starting with a unit square.
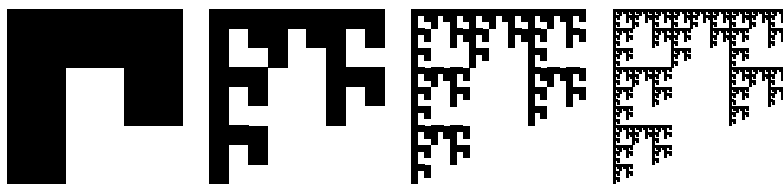
(a) Find the area of the nth stage of the construction.

(b) What is the area of the ultimate figure?

(c) What is the fractal dimension of the ultimate figure?

Question 3a. Find the fixed points of the real function $f(x) = 7 - 2x$.

(b) Find the fixed points of the real function $f(x) = x^2 - 2$.

(c) Are the fixed points of the real function $f(x) = x^2 - 2$ attracting or repelling?

Question 4. Compute the following complex numbers.

(a) $(4 - i)(3 - 2i)$

(b) $(1 + 2i)^2 + (3 - i)$

(c) $\left(\dfrac{1 + i}{\sqrt{2}}\right)^8$

Bonus Question 5. Prove that i is in the Mandelbrot set by showing that the iterates of 0 under the complex function $f(z) = z^2 + i$ eventually repeat, and so are bounded.