What is a statement

Definition ("Def" for short)
A statement is an expression that is either true or false, but not both.

- $4 + 7 = 11$ is a True (T) statement
- $4 + 7 = 2$ is a False (F) statement

The following are not statements because they have no T or F associated with them:

- Find $4 + 7$
- Solve for $x + 7 = 12$

What is a statement

Is each of the following a statement?

- Justin scores an A in Fall 2009.
  It looks like a statement, if Justin scores an A in math and a B in English, the sentence is incomplete, and can be T & F at the same time. Mathematically, we don't consider this to be a statement.
- (1) Justin scored an A in at least one subject Fall 2009.
- (2) Justin scored an A in all subjects Fall 2009.

Both are statements. T/F: Looking up QUEST.

Proving Statements

Using earlier example on Justin:

- To prove (1) is true, only need to find one subject Justin scored an A. To prove (2) is true, we need to check all subjects.
- To disprove (1) (show that it is F), need to check all subject scores not being A. To disprove (2), only need one subject score that is not an A (called a counterexample).

Proving Statements

Consider 3 statements:

A If n is a positive integer then $n^2 + 13$ is not a perfect square.

B If n is a positive integer then $1141n^2 + 1$ is not a perfect square.

C If n is a positive integer then $n^2 + 1$ is not a perfect square.

Is A true?

Try	n	1	2	3	4	5	6
	$n^2 + 13$	14	17	22	29	38	49
							7^2

Since A is false for $n = 6$, A is false.

Proving Statements

- Is B true?
  B is true for $n = 1, \ldots,$ but not for
  $n = 30693385322765657197397207$ (call this n_0).

$$1141n_0^2 + 1 = 1036782394157223963237125215^2$$

- C is true.
  Proof:

$$n^2 < n^2 + 1 < n^2 + 2n + 1 = (n+1)^2$$

  Since n^2 and $(n+1)^2$ are consecutive squares, nothing between them is a square.

How do we prove a statement for all positive integers n?

Notation $\forall n \in \mathbb{P}$

We will learn a method in Chapter 4 later in the course.