Lec 21, July 20, 2010.

Note Title 20/07/2010

For any 2 capacities
are they related by "≤" ∀N
or are they incomparable,
or we don't know?

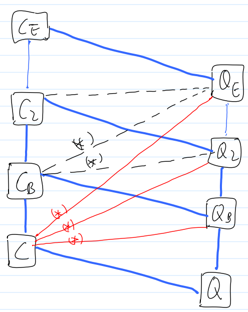
0406086 Bennett, Devetak, Shor, Smolin
0904.4050 Smith, Smolin.

(*) R_d & classical channel.

C small but Q_B Q_2 Q_E all large.

$C_E = 1$ ∴ $Q_E = \frac{1}{2}$

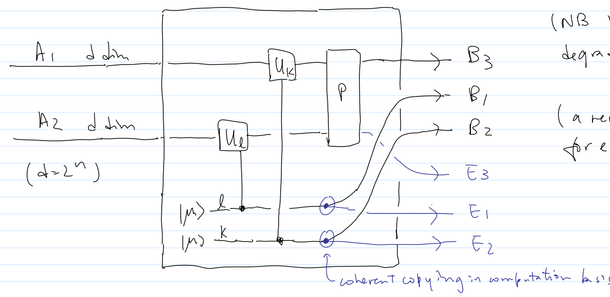
But $C = 1$, $Q \le Q_B \le Q_2 \le Q_E = \frac{1}{2}$



The Rocket channel R_d (iso ext):

(NB Neither degradable / anti deg).

(a new (l, k) is drawn for every use of R_d)



$|\mu\rangle = \frac{1}{\sqrt{|C_n|}} \sum_{l=1}^{|C_n|} |l\rangle$, $\quad P|i\rangle_{A_1}|j\rangle_{A_2} = \omega^{ij} |i\rangle_{A_1}|j\rangle_{A_2}$

C_n: Clifford group on n qubits, ω = primitive dth root of unity

Properties of R_d to be shown / discussed:

① $C(R_d) \le 2$ ∴ $\underbrace{Q(R_d) \le P(R_d) \le C(R_d)}_{\text{general ineq } \forall \text{ channel}} \le 2$

② $Q_E(R_d) \ge \log d$, ∴ $C_E(R_d) \ge 2 \log d$

$P_E(R_d) \ge 2 \log d$

③ $Q^{(1)}(R_d \otimes E_{\frac{1}{2}}) \ge \frac{1}{2} \log d$ ∴ $Q_{ss}(R_d) \ge \frac{1}{2} \log d$

④ $Q_2(R_d) \ge \log d$

$Q_B(R_d) \ge \frac{1}{2} \log d$

$C_B(R_d) \ge \frac{2}{3} \log d$

① The channel is designed to suppress Q & C:

- U_l unknown to sender

  Else sender can input $U_l^{-1}|0\rangle$ to A_2 and $A_1 \to B_3$ will then be a noiseless Q channel, $Q(R_d) \ge \log d$

- U_K unknown to sender

  Else $U_K^{-1}|i\rangle$ for $i = 1 \cdots d$ will emerge as phase × $|i\rangle$ and $A_1 \to B_3$ will be a noiseless C classical, $C(R_d) \ge \log d$

- See 0904.4050 for a proof why $C \le 2$.

  Main idea: ∀ input, averaged over l, k, $B_3^{\otimes n}$ & $E_3^{\otimes n}$ very entangled

  This can be rigorously proved: $\forall \rho \quad S(R_d^{\otimes n}(\rho)) \ge n(\log d - 2)$

  So an upper bound $C(R_d) \le 2$ (even if $\chi^{(1)} \ne C$).

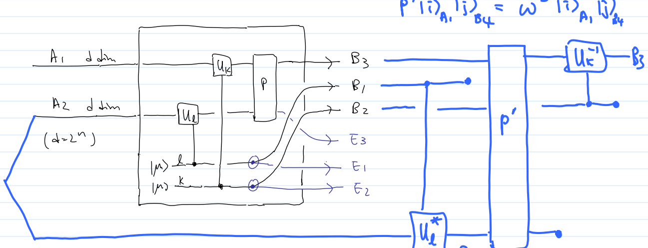
$P_E(R_d) \ge 2 \log d$ ∴ $C_E(R_d) \ge 2 \log d$

② $Q_E(R_d) \ge \log d$:

R_d is made to be "echo-correctable" or "retro correctable"

If A & B can consume $\log d$ ebits

$P'|i\rangle_{A_1}|j\rangle_{B_4} = \omega^{-ij}|i\rangle_{A_1}|j\rangle_{B_4}$



Then $A_1 \to \square \to B_3$ is identity channel on d dims.

∴ $Q_E(R_d) \ge \log d$,

We can take the $A_1 \to B_3$ channel & use another $\log d$ ebits to perform superdense coding.

∴ R_d + $\log d$ ebit $\ge$ $2 \log d$ cbits → — (2a)

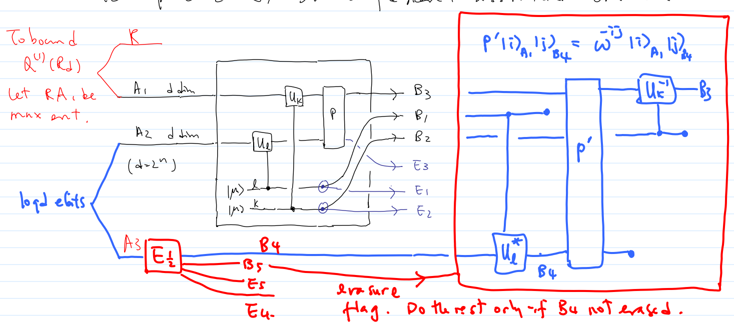
∴ $P_E(R_d) \ge 2 \log d$, $C_E(R_d) \ge 2 \log d$.

(Tempting to say "=").

③ $Q^{(1)}(R_d \otimes E_{\frac12}) \geq \frac12 \log d$.

Idea: use $E_{\frac12}$ to generate log d ebits wp $\frac12$

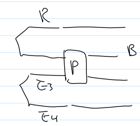
Then proceed w/ entanglement assisted communication.



wp $\frac12$, B_4 not erased. $I_c(R\rangle B_1) = \log d$

wp $\frac12$, B_4 erased, • A_2 & E_4 max ent

• for each l, k (known to Bob & Eve)



and $S(RB_1) = S(B_1) = \log d$

∴ $I_c(R\rangle B_1) = \log d$

(In both cases, other sys with Bob irrelevant in calculating I_c.)

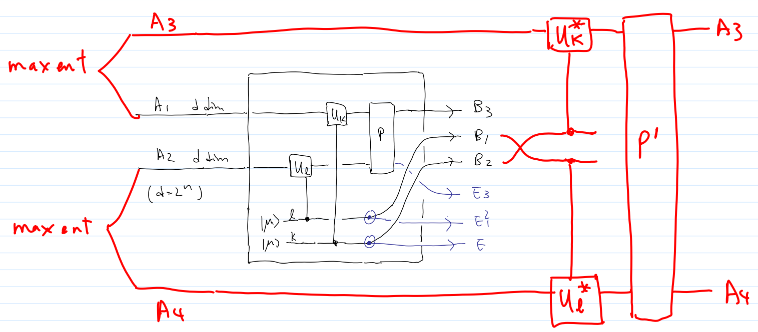
∴ $Q^{(1)}(R_d \otimes E_{\frac12}) \geq \frac12 \log d + \frac12 \cdot 0 = \frac12 \log d$

(Though $Q(R_d) \leq 2$, $Q(E_{\frac12}) = 0$, very superadditive.)

∴ $Q_{ss}(R_d) \geq \frac12 \log d$.

④ $Q_2(R_d) \geq \log d$.

Give free 2-way comm, Bob meas $B_1 B_2$ and send them to Alice. For simplicity, Bob sends $B_1 B_2$ to Alice.



So $A_3 B_3$ max ent (& $E_3 A_4$ max ent).

(R_d + 2 log $|C_n|$ cbits $\geq$ log d ebits) — (4a)

Alice uses free forward CC to teleport log d qubits

(R_d + 2 log $|C_n|$ cbits + 2 log d cbits $\geq$ log d qbits) (4b)

∴ $Q_2(R_d) \geq \log d$.

If only free backward CC is available, we concatenate (4a) & (2a) ie generate log d ebits & run ent assisted protocol

$2R_d + 2\log|C_n|$ cbits $\underset{(4a)}{\geq} R_d + \log d$ ebit $\underset{(2a)}{\geq} \log d$ qbit

∴ $Q_B \geq \frac12 \log d$.

Finally, using (4a) twice & (2a) once, we can do superdense coding:

$3R_d + 4\log|C_n|$ cbits $\underset{(4a)\times 2}{\geq} R_d + 2\log d$ ebit

$\underset{2a}{\geq}$ log d qbit + log d ebit $\underset{SD}{\geq}$ 2 log d cbit.

∴ $C_B(R_d) \geq P_B(R_d) \geq \frac23 \log d$.

$C_2(R_d) \geq \log d$ from $Q_2(R_d) \geq \log d$ but I don't see how to make C_2 larger.