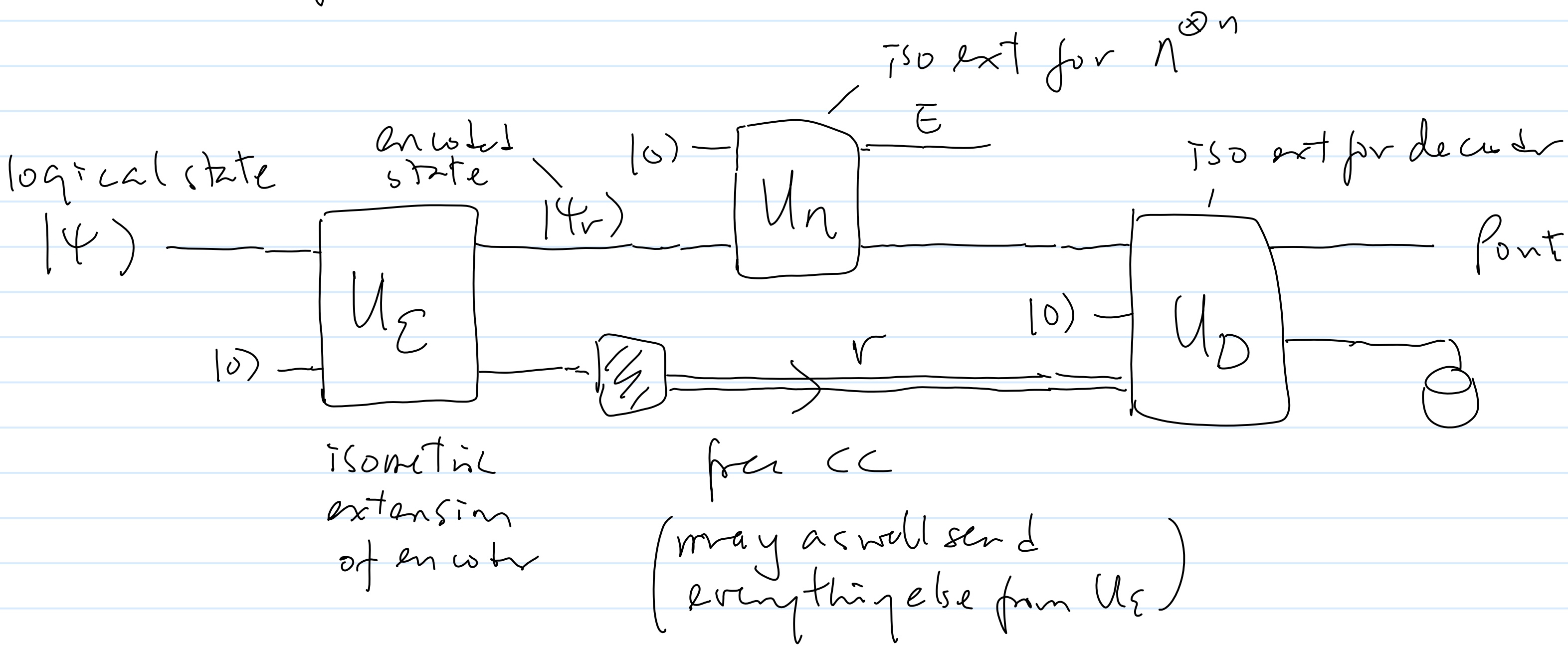
(Left over from lec 14)

What about $Q_{\to}$?

Claim: $\forall \mathcal{N}$ $Q_{\to}(\mathcal{N}) = Q(\mathcal{N})$ (so free forward CC does not increase $Q(\mathcal{N})$).

Pf ideas from BDSW96



① For error to vanish asymptotically, likely outcomes of r has lower error rate

$\therefore \exists r_0$ s.t. for most $|\psi\rangle$, $\rho_{out} \approx |\psi\rangle\langle\psi|$.

Consider that r_0 & restrict to those $|\psi\rangle$'s.

② Then $|\psi\rangle \to |\psi_{r_0}\rangle$ approximately inner product preserving

Else $\rho_{out} \not\approx |\psi\rangle\langle\psi|$.

$\therefore |\psi\rangle \to |\psi_{r_0}\rangle$ can be implemented isometrically, say, by W_{r_0}

$$|\psi\rangle - \boxed{W_{r_0}} \to |\psi_{r_0}\rangle$$

③ Replacing U_E by W_{r_0} makes communicating r_0 unnecessary

- The actual proof relies explicitly on a useful fact:

Thm: Isometric encoding is sufficient (for any encoder, one can replace it by an isometric one w/ slight reduction of fidelity)

See eg 0311037 (Kretschmann & Werner) Sec 5 for detailed proof.

- They also handles all approximation in Sec 6

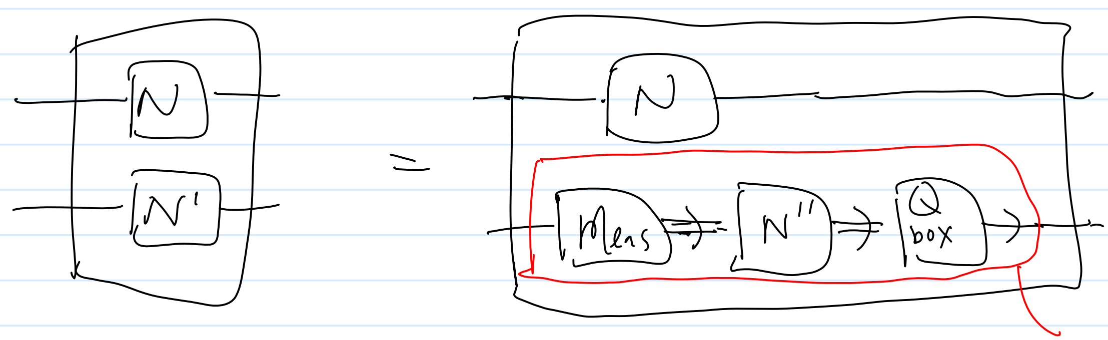
- As a corollary of $Q_{\to}(N) = Q(N)$, they show a stronger result:

$\boxed{\text{Cor 6.2}}$ If N' entanglement breaking then $Q(N \otimes N') = Q(N)$

Pf: Note, $Q_{\to}(N) = Q(N \otimes N'')$

where N'' = perfect classical channel of sufficiently large dim

Ideas: ①



the most general form of ent. breaking channels

② Eq (15) in the paper says

$$Q(N_2 \circ N_1) \leq \min\left(Q(N_2),\ Q(N_1)\right)$$

Together: $Q(N \otimes N') = Q(N \otimes Q_{box} \circ N \otimes N'' \circ N \otimes \mathrm{Meas})$

$\leq Q(N \otimes N'') = Q(N)$.