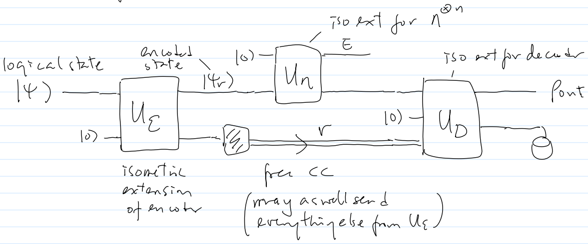
[Left over from lec 14]

What about $Q_\rightarrow$?

Claim: $\forall N$ $Q_\rightarrow(N) = Q(N)$ (so free forward CC does not increase $Q(N)$)

Pf ideas from BDSW96



① For error to vanish asymptotically, likely outcomes of r has lower error rate
$\therefore \exists r_0$ s.t. for most $|\psi\rangle$, $\rho_{out} \approx |\psi\rangle\langle\psi|$.
Consider that r_0 & restrict to those $|\psi\rangle$'s.

② Then $|\psi\rangle \rightarrow |\psi_{r_0}\rangle$ approximately inner product preserving
Else $\rho_{out} \not\approx |\psi\rangle\langle\psi|$.
$\therefore$ $|\psi\rangle \rightarrow |\psi_{r_0}\rangle$ can be implemented isometrically, say, by W_{r_0}

$|\psi\rangle - \boxed{W_{r_0}} - |\psi_{r_0}\rangle$

③ Replacing U_E by W_{r_0} makes communicating r_0 unnecessary

- The actual proof relies explicitly on a useful fact:

Thm: Isometric encoding is sufficient (for any encoder, one can replace it by an isometric one w/ slight reduction of fidelity)

See eg 0311037 (Kretschmann & Werner) Sec 5 for detailed proof.

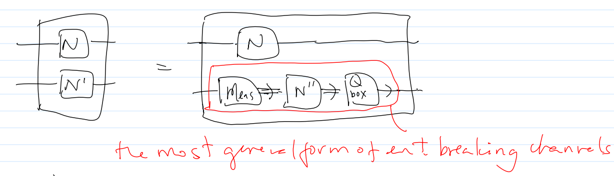
- They also handles all approximation in Sec 6

- As a corollary of $Q_\rightarrow(N) = Q(N)$, they show a stronger result:

$\boxed{\text{Cor 6.2}}$ If N' entanglement breaking then $Q(N \otimes N') = Q(N)$

Pf: Note, $Q_\rightarrow(N) = Q(N \otimes N'')$
where N'' = perfect classical channel of sufficiently large dim

Ideas: ①



② Eq (15) in the paper says
$$Q(N_2 \circ N_1) \leq \min(Q(N_2), Q(N_1))$$

Together: $Q(N \otimes N') = Q(N \otimes Q_{box} \circ N \otimes N'' \circ N \otimes \text{Meas})$
$\leq Q(N \otimes N'') = Q(N)$.