University of Waterloo STAT 332 – W. H. Cherry

Review of Probability Results

The following results from second-year probability and statistics courses may be needed and you should be familiar with them.
Note that Y and Z denote random variables; a, b, c and d are constants.
The following abbreviations are used: 'E' for *expectation*, '*s.d.*' for *standard deviation*, '*var*' for *variance*, and '*cov*' for *covariance*.

1. For a *discrete* random variable Y with probability function $f(y)$, the mean of Y is: $E(Y) = \sum_y y \bullet f(y)$;

   for a *continuous* random variable Y with probability density function $f(y)$, the mean of Y is: $E(Y) = \int_{-\infty}^{\infty} y \bullet f(y)dy$;

   $s.d.(Y) \equiv \sqrt{var(Y)} = \sqrt{E[Y - E(Y)]^2} = \sqrt{E(Y^2) - [E(Y)]^2}$.

2. $cov(Y, Z) = E\{[Y - E(Y)][Z - E(Z)]\} = E(YZ) - E(Y) \bullet E(Z)$.

3. $E(a + bY) = a + bE(Y)$;

   $s.d.(a + bY) = |b|\, s.d.(Y)$.

4. $cov(a + bY, c + dZ) = bd \bullet cov(Y, Z)$.

5. $E(Y \pm Z) = E(Y) \pm E(Z)$;

   $s.d.(Y \pm Z) = \sqrt{[s.d.(Y)]^2 + [s.d.(Z)]^2 \pm 2cov(Y, Z)}$

   $= \sqrt{[s.d.(Y)]^2 + [s.d.(Z)]^2}$ if Y and Z are *probabilistically independent.*

6. $E(bY \pm dZ) = bE(Y) \pm dE(Z)$;

   $s.d.(bY \pm dZ) = \sqrt{b^2 \bullet [s.d.(Y)]^2 + d^2 \bullet [s.d.(Z)]^2 \pm 2bd \bullet cov(Y, Z)}$

   $= \sqrt{b^2 \bullet [s.d.(Y)]^2 + d^2 \bullet [s.d.(Z)]^2}$ if Y and Z are *probabilistically independent.*

7. $E(\sum_i b_i Y_i) = \sum_i b_i E(Y_i)$;

   $s.d.(\sum_i b_i Y_i) = \sqrt{\sum_i b_i^2 \bullet [s.d.(Y_i)]^2 + \sum_i \sum_{j \neq i} b_i b_j \bullet cov(Y_i, Y_j)}$

   $= \sqrt{\sum_i b_i^2 \bullet [s.d.(Y_i)]^2}$ if the Y_i are mutually *probabilistically independent.*

8. $cov(\sum_i b_i Y_i, \sum_j d_j Z_j) = \sum_i \sum_j b_i d_j \bullet cov(Y_i, Z_j)$.

9. A linear function of a normally distributed random variable is normally distributed.

10. A linear combination of normally distributed random variables is normally distributed.

1995-04-20

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