Figure 2.1. SURVEY SAMPLING: An Introduction

Sample survey:
Defining
Specifying
Selecting
Responding
Measuring
Estimating

A **sample survey** involves making inferences about a population on the basis of information obtained from a sample (*i.e.*, a subset) selected from the population; inherent in this description of *sampling* are the processes of *defining* the target population appropriately for the Question(s) to be answered, *specifying* the study population (*e.g.*, by a suitable **frame**), *selecting* units (a unit is one or more elements) and *estimating* attributes, which entails *measuring* response (and perhaps explanatory) variate(s) for each unit selected. The alternative to a sample survey is a **census**, in which *every* element of the population is investigated.

- ∗ **Target population:** the group of elements to which the investigator(s) want Answer(s) to the Question(s) to apply.
- ∗ **Study population:** a group of elements *available* to an investigation.
- ∗ **Respondent population:** those elements of the study population that *would* provide the data requested under the incentives for response offered in the investigation;
- ∗ **Non-respondent population:** those elements of the study population that would *not* provide the data requested under the incentives for response offered in the investigation.

Census:
Defining
Specifying
Responding
Measuring

There is further discussion of the respondent and non-respondent populations in Section 3 overleaf on pages 2.4 and 2.5.

The choice between a sample survey and a census is usually a trade-off between their advantages and disadvantages. The *advantages* of sample surveys are that they:

- + use fewer *resources* (money and/or time);
- + are usually shorter in duration and so provide more *timely* Answer(s) to Question(s);
- + are the only feasible option when measuring is *destructive*; *e.g.*, crash tests on cars, test firing of camera flash bulbs and gun cartridges, measuring cigarette tar and nicotine levels and bursting pressures of plastic bags and condoms.

The main *dis*advantages of sample surveys are that:

- – there is inherent *uncertainty* in using a sample attribute as a basis for *estimating* a population attribute;
- – they usually do not provide information about *every* segment (or subgroup) of the population being investigated – see, for example, the comment at the bottom of the first column, and continued at the top of the middle column, on page 2.31 in Figure 2.2f.

1. Terminology

We distinguish an *element* from a *unit.*

- **Elements** are the entities that make up a population; for example, a person is an element of the population of Canadians, but we recognize that many populations in data-based investigating have non-human or *in*animate elements.
- **Units** are the entities *selected* for the sample; a unit may be one element (*e.g.*, a person) or *more than* one (*e.g.*, a household).

In multi-stage selecting (discussed below), we speak of *primary* selecting units (PSUs) selected at the *first* stage, *secondary* selecting units (SSUs) selected at the *second* stage, and so on.

Population < Unstratified / Stratified
Number of stages < One-stage / Multi-stage
Last stage units < Elements / Clusters
Selecting method < Probability / Other

Major components of the Plan for a sample survey include the four matters summarized at the right; details are as follows.

- **Stratifying** is a process of subdividing a population into groups called **strata** before the units are selected; strata must be defined in such a way that every population unit belongs to *one and only one* stratum. Units for the sample are selected from *each* of the strata.
- Units can be selected from the population in one or more **stages**, the level of aggregation of the units *de*creasing through successive stages; for Canada, for example, a *five*-stage Plan could involve selecting *provinces* within the country, *counties* within the provinces selected, *urban and rural areas* within these counties, *households* within the selected areas, and a *person* within each of these households.
  - – A sample survey to obtain *household* data would involve only the first *four* of these stages.
- The *last* stage of a multi-stage sample survey (or a *one*-stage survey) can select units which are either *individual* population elements or *groups* of elements (called *clusters*); obviously, stages *before* the last can *only* select *groups* of elements.
- A variety of processes is used to *select* units at each stage; the basis of the statistical theory for estimating population attributes from sample data is *probability selecting*, in which each population unit has a *known* probability of being selected for the sample.

2. Selecting

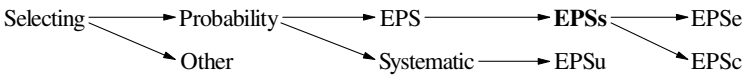
We distinguish the following processes for *selecting* units for sample surveys:

- probability selecting;
- accessibility selecting;
- haphazard selecting;
- volunteer selecting;
- convenience selecting;
- quota selecting;
- judgement selecting.

In *probability selecting* (also called 'probability sampling'), every unit of the study population has a *known* probability of being selected. Sample survey Plans are often developed so that, as far as practicable, every unit of the population has (approximately) the *same* selection probability; this is the case for *equiprobable selecting* (EPS) [but see Note 2 overleaf on page 2.4].

(*continued overleaf*)

We need to recognize *which entities* are equiprobable in EPS – all possible *samples of a given size* (EPSs) or all population *units* (EPSu) or all population *elements* (EPSe) or all population *clusters* (EPSc); the schema below summarizes these possibilities. The meaning of EPS in *these* Course Materials [and, elsewhere, of 'simple random selecting (SRS)'] is shown in **bold** type in the schema.



NOTES: 1. The foregoing discussion of EPS is based on equiprobable selecting with*out* replacement (EPSWOR); survey sampling and these Course Materials have little concern with selecting *with* replacement (EPSWIR). *We* usually omit *WOR* – its inclusion in some titles (*e.g.*, in STAT 220 Figure 8.11 and STAT 332 Figure 2.10) is an occasional reminder.

2. For a respondent population frame of four units a, b, c, d, six samples of size 2 (as listed at the right) can be selected by EPS but only *two* by systematic selecting (SYS); these two samples are *equally* probable if the selecting starting point is chosen equiprobably. Thus, under *both* selecting processes, each unit has an inclusion probability of one-half. This reminds us that EPS implies equal *unit* inclusion probabilities but the converse is *not* true – here, systematic selecting with *equal* inclusion probabilities is *not* EPS because it cannot select four of the samples possible under EPS.

EPS	SYS
ab	ac
ac	bd
ad	
bc	
bd	
cd	

3. A phrase commonly used in texts dealing with survey sampling is *simple random selecting* [or, regrettably, *simple random* ***sampling***] (SRS); in the terminology of these Course Materials, it is equivalent to *EPS from an unstratified population. We* use this longer phrase because it is more evocative of the actual selecting process.

4. If there are **N** units in the respondent population and there are n units in the sample, then: $f = \frac{n}{N}$ is called the **sampling fraction.** -----(2.1.1)
[*f* is the probability any respondent population unit is included in a sample obtained by EPS.]

Probability selecting has two advantages.

- It is the basis of statistical theory for survey sampling; *e.g.*, it is the basis of expressions for confidence intervals for respondent population attributes.
- For estimating attributes like averages, totals, proportions and frequencies, *under repetition* it eliminates sampling *inaccuracy*.
  - Zero sampling inaccuracy under repetition has limited implication for the magnitude of sample error – it only marginally reduces the risk of sample error large enough to impose unacceptable limitation on Answer(s).
  - If a sample, *after* it has been selected, is found to have sample error that imposes unacceptable limitation on an Answer, it is usually impossible to determine, from an examination of the *sample itself*, the extent to which the sample error is due to a flawed selecting process or to chance; instead, it is necessary to know the *process* by which the sample was selected.

Related matters are:

- The statistical theory of survey sampling usually deals *only* with the uncertainty that arises because of the sample data are inherently *incomplete*; it seldom takes account of *other* sources of uncertainty (sometimes called *non-sample errors* – see Figure 3.2) such as study error, non-response error, data processing mistakes, etc. (See Note 9 at the bottom of the facing page 2.5 and also see Section 7 on pages 6.6 and 6.7 in Figure 6.1 of the STAT 220 Course Materials.)
- *Non*-probability selecting processes can yield Answers with acceptable limitations in some investigations, but the results of (probability) selecting theory (*e.g.*, to obtain confidence intervals for respondent population attributes) do *not* apply to them.
  - A common *misuse* of statistical methods is to give confidence intervals based on sample survey data obtained using *any* of the six *non*-probability selecting processes listed overleaf in Section 2 near the bottom of page 2.3.
  - The experience of sample survey statisticians over several decades provides compelling evidence that, when human judgement plays a significant role in sample selecting (as is often the case in non-probability processes), sample error is more likely to impose *un*acceptable limitation on Answers, particularly for Questions with a *descriptive* aspect.
  - The names of the six *non*-probability selecting processes listed overleaf on page 2.3 do not necessarily specify a *unique* selecting method – the first two methods overlap and the first five involve some degree of 'accessibility' and/or 'convenience'.

    Haphazard selecting is sometimes ***wrongly*** equated with 'random' selecting; *i.e.*, with our *equiprobable* selecting.

    Quota selecting is a similar idea to **covering** – see the top of page 5.24 in Figure 5.7 of the STAT 231 Course Materials.

    Volunteer selecting is *not* to be confused with **volunteer** (or **voluntary**) **response**, a phrase sometimes used to indicate that *human* elements can (usually) *choose* whether to respond, *i.e.*, whether to provide the requested data; a separate (measuring) issue is whether these responses are correct or truthful (see Note 68 on page 5.62 in STAT 231 Figure 5.7).

3. Responding

Many data-based investigations encounter the difficulty that not all the data called for in the Plan are acquired – this is the general problem of *missing data*. For *human* populations, this matter is usually referred to as *non-response* and it can impose *un*acceptable limitations on Answer(s). A framework for discussing non-response is shown in the diagram at the upper right of the facing page 2.5, where the *study* population (represented by the outer area) is made up of the *respondent* population and the *non-respondent* population. The set of units selected from the study population is the **selection**, and comprises the **sample**

Figure 2.1. SURVEY SAMPLING: An Introduction (continued)

(from the respondent population) and the **non-respondents** (from the *non*-respondent population). The diagram has *two* categories of symbols:

– $\mathbf{N}$s and ns refer to *numbers of (elements or) units*;

– $\bar{\mathbf{Y}}$s and $\bar{y}$s are *averages* of a response variate $\mathbf{Y}$ of the (elements or) units.

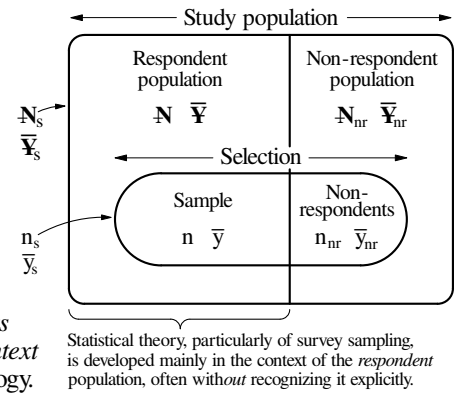
The relationships among the numbers of (elements or) units are:

Study population = Respondent population + Non-respondent population

$\mathbf{N}_s = \mathbf{N} + \mathbf{N}_{nr}$

Selection = Sample + Non-respondents

$n_s = n + n_{nr}$



Statistical theory, particularly of survey sampling, is developed mainly in the context of the *respondent* population, often with*out* recognizing it explicitly.

NOTES: 5. In these Course Materials, use of '*sample*' to refer only to *respondents* is *seldom* followed elsewhere; it can be confusing when only the *context* indicates whether 'sample' means *selection* or *sample* of our terminology.

- Phrases like *intended sample* and *achieved sample* are also used in some places to convey the same distinction as our *selection* and *sample*.

6. The symbols $\mathbf{N}$ (for the number of respondent population elements or units) and n (for the number of units in the sample) could usefully be written as $\mathbf{N}_R$ and n_r as a reminder that they refer to *respondents*. However, the *un*subscripted symbols $\mathbf{N}$ and n are in such widespread use that it is unrealistic to expect to change them; in addition, there would be the inconvenience of a subscript on two frequently-used symbols.

7. A major Plan consideration in a sample survey of a *human* population is how to offer (as specified in the *sampling protocol*) adequate *incentives for response* (*e.g.*, proper presentation of the survey, suitable follow-up, rewards for response); the goal of the incentives is to achieve as *high* a response *rate* as possible. An illustration of why a high response rate is important is provided in Table 2.1.1 below.

- "The first column of Table 2.1.1 at the right lists seven levels of non-response for a survey question with a *Yes* or *No* answer. The entries in the second and third columns of the Table show *intervals* of possible percentages of *Yes* answers among *all* the units *selected*. The second column is based on the assumption that, in the *sample*, half the answers are *Yes* and half are *No*, regardless of the non-response rate; for the third column, it is assumed that 90% of the *sample* answer *Yes* and 10% answer *No*, regardless of the non-response rate.

Table 2.1.1 PERCENTAGE OF *Yes* ANSWERS AMONG ALL UNITS SELECTED

NON-RESPONSE RATE	50% *Yes* in Sample	90% *Yes* in Sample
0%	50%	90%
5	47.5 – 52.5	85.5 – 90.5
10	45 – 55	81 – 91
25	37.5 – 62.5	67.5 – 92.5
50	25 – 75	45 – 95
75	12.5 – 87.5	22.5 – 97.5
90	5 – 95	9 – 99

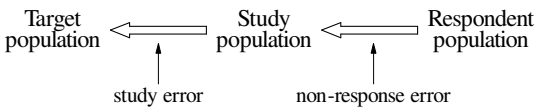
We see that every percentage point of non-response adds a percentage point to the range (*i.e.*, to the uncertainty) for the proportion of possible *Yes* answers in the *selection*. Also, for 90% *Yes* in the sample, the *centres* of the ranges differ progressively more from 90% as we go down Table 2.1.1; this reflects *non-responding inaccuracy* arising from a situation where *Yes* and *No* are not equally likely.

8. Two matters about the separation of the study population into the respondent and *non*-respondent populations are:

- Which elements of the study population fall in which of the two populations depends on the *incentives* offered for response – different incentives would presumably, in general, result in *different* sets of the study population elements in the two populations. For *given* incentives for response (as specified in the sampling protocol of a *particular* survey), *we* usually assume that a given element will *always* make the *same* decision about whether or not to respond; this *deterministic* view of the response decision is why *we* usually limit discussion to non-response *error*, rather than going on to non-responding *inaccuracy*.
- The respondent and non-respondent populations are *conceptual* in the sense that we only *encounter* subsets of them (as the sample and the non-respondents); if a unit is *not* included in the selection, we generally do *not* know (and do not *need* to know) to which of the two populations it belongs.

9. In addition to *sample* error and *measurement* error when estimating *respondent* population attributes, the inference *back* from these estimates to plausible values for *study* population attributes and, hence, for those of the *target* population is subject to the effects of error from two *other* sources:

- *non-response* error (due to differing attribute values in the respondent and study populations);
- *study* error (due to differing attribute values in the study and target populations);



the schema at the right above shows this matter pictorially. All *four* error categories impose limitations on Answers.

(*continued overleaf*)

4. Measuring

It is convenient to distinguish *four* components of a *measuring process*:

- the measuring instrument or gauge;
- the operator;
- the measuring protocol;
- the element (or unit) measured.

In the context of a sample survey, the measuring *instrument* is the questionnaire. It is curious that, when sample surveys are carried out, investigators who would *never* undertake to assemble the types of instrumentation typically used in a laboratory (*e.g.*, balances, spectrophotometers) approach the task of developing a *questionnaire* (*i.e.*, a *sample survey* measuring instrument) with what seems to be little or no recognition of either the difficulty or the importance of doing so successfully.

5. Estimating

In *our* discussion of survey sampling, we are concerned with estimating five types of attributes:

- averages;
- totals;
- proportions;
- frequencies;
- ratios of averages.

For averages and proportions, the corresponding sample attribute (or 'statistic') is generally used to estimate the respondent population attribute, but the sample total and sample frequency do *not* estimate the respondent population total or frequency; ratios of averages are more complicated to estimate.

- ○ We speak of *estimating* from the data obtained from a *sample*, but of *imputing* to deal with *missing* data for *non-respondents*.
  - – An imputed value for a non-response is (hopefully) a 'best guess' of the missing response variate data based, for example, on known values of relevant *explanatory* variate(s) of the unit concerned; imputed values are included to facilitate data analysis. Rarely does imputing meaningfully reduce the severity of limitations on Answers by reducing incompleteness of data.
- ○ The term 'statistic', beloved of introductory texts, is *avoided* in these Material; instead, *we* use 'estimator' or 'estimate' as appropriate. For us, 'population parameter' and 'sample statistic' are 'population attribute' and 'sample attribute' – *models* have parameters.

6. 'Good' Sample Surveys

A summary of Plan requirements for a good sample survey (*i.e.*, one with *acceptable* limitations on Answers) is as follows.

- ⊙ A well-presented questionnaire, with unambiguous and answerable questions, which is properly administered and which gathers the information needed to address properly the Question(s) to be answered.
- ⊙ A sample of adequate size, obtained by a probability selecting method.
- ⊙ A high response (or low *non*-response) rate.
- ⊙ Correct use of statistical theory, together with accurate data processing and clear and complete presentation of the Answer(s).

Deficiencies in any *one* of these four Plan components, *even if all other components are adequate*, can impose unacceptable limitations on the Answer(s) obtained from the survey.

A *report* of a sample survey should, as a *minimum* requirement, provide the reader with enough information about the Plan to make clear the following *four* matters:

- ∗ What the target and the study populations were.
- ∗ What the non-response rate was.
- ∗ What process was used to select the units.
- ∗ What the sample size was.

The first question is concerned with *study* error, the second with *non-response* error and the third and fourth with *sample* error. The importance of *sample size* is that, *for fixed data quality* and assuming the same information content per observation, the *larger* the amount of data the *more* precise the Answer(s) the data can yield. However, it is almost always (extremely) *difficult* to maintain high quality in (very) large data sets, one reason why a sample survey is usually preferred over a census.

Assessment of *measurement* error requires, as a minimum, a copy of the questionnaire (*e.g.*, in an appendix of the report).

NOTE: 10. Despite a central concern with survey sampling, these Course Materials seldom uses the word **sampling** [except in the (established) phrases *survey sampling*, *sampling protocol*, *sampling inaccuracy*, *sampling imprecision* and *sampling fraction*] for three reasons:

- the general use of the word encompasses the four processes of *selecting*, *responding*, *measuring* and *estimating*, which are best *explicitly* kept as *separate* entities and considered *individually*;
- the specific use is basically a synonym for the process of *selecting*, a word whose clarity can be lost when it is instead called 'sampling';
- some uses of the word could be taken as implying that sampling (in the sense of reasoning from data derived from a *subset* of some population) is primarily the domain of sample surveys [*i.e.*, investigations with an observational Plan to answer Question(s) with a descriptive aspect] whereas, in reality, essentially *all* types of investigations [including those with observational or experimental Plans to answer Question(s) with a descriptive or causative aspect] take only a *sample* – that is, a census is *rare* in data-based investigating.

In other sources of information about survey sampling, you are likely to encounter the word *sampling*; when you do, you should identify the sense in which it is being used.