# Figure 4.3. ANALYSIS STAGE: Histograms 2

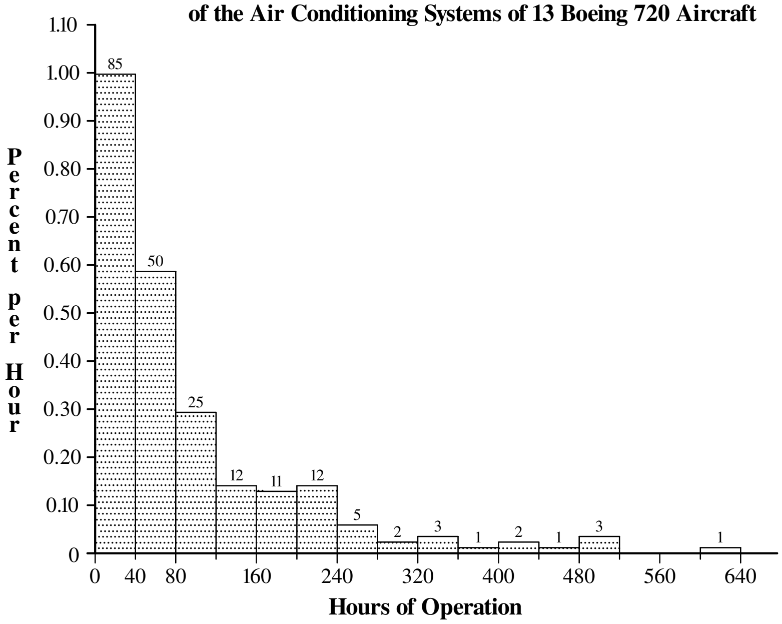
The data in the following table are failure times of the air conditioning systems of a fleet of 13 Boeing 720 jet aircraft; the figures for each plane are the numbers of hours of operation between *successive* failures of its air conditioning system. After around 2,000 hours of operation, each aircraft was due to receive a major overhaul (denoted '*MO*' in the table); if a time interval between successive failures of the air conditioning system of a plane included a major overhaul, the length of the interval has been *omitted* from the data set, because of the possible effect of the overhaul on the magnitude of the interval.

| Plane | Hours of operation between successive failures of the air conditioning system | Data summaries: n | $\sum_{j=1}^{n} y_j$ | $\sum_{j=1}^{n} y_j^2$ |
|---|---|---|---|---|
| 7907 | 194 15 41 29 33 181 | 6 | 493 | 74,233 |
| 7908 | 413 14 58 37 100 65 9 169 447 184 36 201 118 *MO* 34 31 18 18 67 57 62 7 22 34 | 23 | 2,201 | 523,687 |
| 7909 | 90 10 60 186 61 49 14 24 56 20 79 84 44 59 29 118 25 156 310 76 26 44 23 62 *MO* 130 208 70 101 208 | 29 | 2,422 | 342,656 |
| 7910 | 74 57 48 29 502 12 70 21 29 386 59 27 *MO* 153 26 326 | 15 | 1,819 | 553,767 |
| 7911 | 55 320 56 104 220 239 47 246 176 182 33 *MO* 15 104 35 | 14 | 1,832 | 365,078 |
| 7912 | 23 261 87 7 120 14 62 47 225 71 246 21 42 20 5 12 120 11 3 14 71 11 14 11 16 90 1 16 52 95 | 30 | 1,788 | 256,420 |
| 7913 | 97 51 11 4 141 18 142 68 77 80 1 16 106 206 82 54 31 216 46 111 39 63 18 191 18 163 24 | 27 | 2,074 | 264,856 |
| 7914 | 50 44 102 72 22 39 3 15 197 188 79 88 46 5 5 36 22 139 210 97 30 23 13 14 | 24 | 1,539 | 188,971 |
| 7915 | 359 9 12 270 603 3 104 2 438 | 9 | 1,800 | 768,288 |
| 7916 | 50 254 5 283 35 12 | 6 | 639 | 148,499 |
| 7917 | 130 493 | 2 | 623 | 259,949 |
| 8044 | 487 18 100 7 98 5 85 91 43 230 3 130 | 12 | 1,297 | 344,335 |
| 8045 | 102 209 14 57 54 32 67 59 134 152 27 14 230 66 61 34 | 16 | 1,312 | 173,558 |
| | **All planes:** | 213 | 19,839 | 4,264,297 |

A frequency table and a histogram of these data, using an interval width of 40 hours, are as follows:

| Class Interval | Frequency No. | Frequency % | Density (%/hour) |
|---|---|---|---|
| [ 0, 40) | 85 | 39.91 | 0.9977 |
| [ 40, 80) | 50 | 23.47 | 0.5869 |
| [ 80, 120) | 25 | 11.74 | 0.2934 |
| [120, 160) | 12 | 5.63 | 0.1408 |
| [160, 200) | 11 | 5.16 | 0.1291 |
| [200, 240) | 12 | 5.63 | 0.1408 |
| [240, 280) | 5 | 2.35 | 0.0587 |
| [280, 320) | 2 | 0.94 | 0.0235 |
| [320, 360) | 3 | 1.41 | 0.0352 |
| [360, 400) | 1 | 0.47 | 0.0117 |
| [400, 440) | 2 | 0.94 | 0.0235 |
| [440, 480) | 1 | 0.47 | 0.0117 |
| [480, 520) | 3 | 1.41 | 0.0352 |
| [520, 560) | 0 | 0 | 0 |
| [520, 560) | 0 | 0 | 0 |
| [600, 640) | 1 | 0.47 | 0.0117 |
| | 213 | 100.00 | |

**Hours of Operation Between Successive Failures of the Air Conditioning Systems of 13 Boeing 720 Aircraft**




**DATA SOURCE:** Proschan, F., *Technometrics* **5**(3): 376 (August, 1963).

(*continued overleaf*)

**NOTE:** The order statistic for the 213 observations tabulated overleaf on page 4.5 is:

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 3 | 3 | 3 | 4 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 7 | 9 | 9 | 10 | 11 | 11 | 11 | 11 | 12 | 12 |
| 12 | 12 | 13 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 15 | 15 | 15 | 16 | 16 | 16 | 18 | 18 | 18 | 18 | 18 | 18 | 20 | 20 |
| 21 | 21 | 22 | 22 | 22 | 23 | 23 | 23 | 24 | 24 | 25 | 26 | 26 | 27 | 27 | 29 | 29 | 29 | 29 | 30 | 31 | 31 | 32 | 33 | 33 |
| 34 | 34 | 34 | 35 | 35 | 36 | 36 | 37 | 39 | 39 | 41 | 42 | 43 | 44 | 44 | 44 | 46 | 46 | 47 | 47 | 48 | 49 | 50 | 50 | 51 |
| 52 | 54 | 54 | 55 | 56 | 56 | 57 | 57 | 57 | 58 | 59 | 59 | 59 | 60 | 61 | 61 | 62 | 62 | 62 | 63 | 65 | 66 | 67 | 67 | 68 |
| 70 | 70 | 71 | 71 | 72 | 74 | 76 | 77 | 79 | 79 | 80 | 82 | 84 | 85 | 87 | 88 | 90 | 90 | 91 | 95 | 97 | 97 | 98 | 100 | 100 |
| 101 | 102 | 102 | 104 | 104 | 104 | 106 | 111 | 118 | 118 | 120 | 120 | 130 | 130 | 130 | 134 | 139 | 141 | 142 | 152 | 153 | 156 | 163 | 169 | 176 |
| 181 | 182 | 184 | 186 | 188 | 191 | 194 | 197 | 201 | 206 | 208 | 208 | 209 | 210 | 216 | 220 | 225 | 230 | 230 | 239 | 246 | 246 | 254 | 261 | 270 |
| 283 | 310 | 320 | 326 | 359 | 386 | 413 | 438 | 447 | 487 | 493 | 502 | 603 | | | | | | | | | | | | |

1 Suggest plausible reason(s) why the data tabulated overleaf on page 4.5 might have been collected. [In the publication in *Technometrics*, Frank Proschan gives his affiliation as *Boeing Scientific Research Laboratories, Seattle, Washington.*]

2 Outline the principal feature(s) of the data set, on the basis of the information provided overleaf on page 4.5.

- Suggest a reasonable *probability model* for the distribution of the data; briefly justify your choice.
  - Calculate the *average* and the *standard deviation* for the data set and comment briefly on how the values you find impinge on your suggestion for a probability model.

3 Find the *median* for the data set and account briefly for how its value compares with that of the average.

4 On the basis of the information provided in this Figure:

- outline what can reasonably be concluded about the reliability of the air conditioning systems in the different planes;
- outline what can reasonably be concluded about the effect of a major overhaul on air conditioning system reliability;
- comment briefly on the statement: *More than a third of the observed failure times are less than 40 hours, which is less than* half *the overall average failure time of about 93 hours; this shows that the air conditioning systems on these aircraft are* very *unreliable.*

**REFERENCE**: Derman, C., Gleser, L.J. and Olkin, I.: *A Guide to Probability Theory and Its Applications.* Holt, Rinehart and Winston, Inc., New York, 1973, also discuss these data on pages 361-365.

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