STATISTICS 230 – Tutorial 1 Problems

T1 – 1. A bus with two passengers will be making 5 stops. Each passenger picks a stop at random, independently of the other passenger, and gets off. It is possible that both passengers select the same stop.

(a) Describe a 25-point sample space for this situation.

(b) Assuming the sample points to be equally probable, find the probabilities of the following events:
A: "no passengers get off at the first stop";
B: "no passengers get off at the first *two* stops";
C: "the passengers get off at *different* stops";

(c) Find the probabilities of events A, B and C if there are 2 passengers and n stops ($n \geq 3$).

(d) Find the probabilities of events A, B and C if there are r passengers and n stops ($n > r$).

T1 – 2. A club has a membership of 10 couples. A committee of 5 people is picked at random. Find the probability the committee has (exactly) one couple.

T1 – 3. From a set of $2n+1$ consecutively numbered tickets, three are selected at random without replacement. Find the probability that the numbers of the tickets form an arithmetic progression. [The *order* in which the tickets are selected does *not* matter.]

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STATISTICS 230 – Tutorial 2 Problems

T2 – 1. Show that: $1^2\binom{n}{1} + 2^2\binom{n}{2} + 3^2\binom{n}{3} + \ldots.. + n^2\binom{n}{n} = n(n+1)2^{n-2}$

HINT: $x^2 = x(x-1) + x$.

T2 – 2. Balls numbered 1, 2, ..., 20 are placed in an urn and one ball is drawn at random. Show each of the following events on a Venn diagram and find their probabilities:

A: "the number on the ball is divisible by 2"; C: "the number on the ball is prime";

B: "the number on the ball is divisible by 5"; D: "the number on the ball is odd."

Then find the probability of each of the following events, which are *combinations* of the previous four events:

$A \cup B$; $A \cap \overline{B}$; $A \cup D$; $A \cap D$; $(A \cup B) \cap (C \cup D)$; $(A \cap B) \cup (C \cap D)$.

T2 – 3. When A and B play a game, the odds that A wins are 2 to 1. If they play two games and games are modelled as being probabilistically independent, what are the odds that A wins them both?

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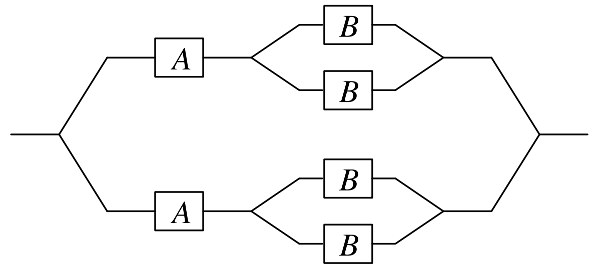
STATISTICS 230 – Tutorial 3 Problems

T3 – 1. An electrical network has switches connected as shown at the right. Switches operate independently and the probability a switch is closed is 0.8 for type A and 0.6 for type B. What is the probability that there is a complete path through the network?



T3 – 2. In a three-cornered duel, A, B and C successively shoot at any other combatant until only one remains. The probability of 'success' on any shot is 0.3 for A, 0.5 for B and 1 for C, and shots can be modelled as being probabilistically independent. Each 'player' adopts the best survival strategy, including possibly an intentional miss. Find the probabilities of survival for A, B and C.

T3 – 3. There are two diagnostic tests for a disease. Among people who have the disease, 10% give negative results on the first test and, independently of this, 5% give negative results on the second test. Among those who do *not* have the disease, 80% give negative results on the first test and, independently, 70% give negative results on the second test. Twenty per cent of those tested have the disease.

(a) If both tests are *negative*, what is the probability that the person tested has the disease?

(b) If both tests are *positive*, what is the probability that the person tested has the disease?

(c) If the first test gives a positive result, what is the probability that the second test will also be positive?

STATISTICS 230 – Tutorial 4 Problems

T4 – 1. Let random variable X have a Poisson distribution with mean μ. Find the probability that X is even.

HINT: Compare the series expansions of e^{μ} and $e^{-\mu}$.

T4 – 2. A batch of g good lightbulbs has become mixed up with a batch of b bad ones. Bulbs are randomly selected one at a time without replacement and tested until all b bad ones have been found. Find the probability function of random variable N, the total number of bulbs tested.

T4 – 3. Digits are chosen at random with replacement from 0, 1, ..., 9. If random variable N is the number of digits that must be chosen in order to obtain at least one 0 and at least one 1, find the probability function of N.

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STATISTICS 230 – Tutorial 5 Problems

T5 – 1. Flaws in the plating of standard-sized sheets of metal occur at random over the entire area being plated, and 67% of the sheets produced have no flaws.

(a) Ten sheets are taken from the assembly line; what is the probability that exactly 3 of them contain flaws?

(b) Sheets are taken from the assembly line until 7 without flaws have been obtained; what is the probability that exactly 10 sheets will be selected?

(c) What percentage of the sheets plated will have more than one flaw?

T5 – 2. Suppose that the number of colds a person gets per year can be modelled by a Poisson distribution with an average of 3. For two-thirds of the population, taking one gram of vitamin C daily reduces this average number of colds to 2, while for the other one-third of the population, vitamin C has no effect. What is the probability a person who takes 1 gram of vitamin C daily and gets one cold in a year is someone who actually benefits from vitamin C?

T5 – 3. Suppose that the number of eggs laid by a female robin can be modelled by a Poisson distribution with mean μ, and that each egg has probability p of hatching, independently of other eggs. Find the probability distribution of the number of offspring.

STATISTICS 230 – Tutorial 6 Problems

T6 – 1. Forty percent of the eggs produced by a flock of hens are classified as *small*, 40% are *medium*, and 20% are *large*. Eggs are chosen at random from a large batch of the eggs until k large eggs have been obtained. If random variable X is the number of small, and random variable Y the number of medium, eggs obtained, find the joint probability function $f(x, y)$.

T6 – 2. Let X and Y be discrete random variables defined on the same sample space, with joint probability function as shown at the right.

$$f(x, y) = p^{x+y}(1-p)^2 \quad \text{for } x = 0, 1, 2, \ldots \quad y = 0, 1, 2, \ldots$$

(a) Derive the p.f. of the total $T \equiv X + Y$.

(b) Find the probability of the event $X = Y$.

(c) Find the conditional p.f. of X given that $T = 4$.

T6 – 3. An urn contains **N** balls numbered 1, 2, ..., **N**. A random sample of n balls is chosen *with* replacement. If variates X and Y represent the largest and smallest numbers drawn, give an expression for each of the following probabilities:

$$\Pr(X \le x); \qquad \Pr(Y > y); \qquad \Pr(X \le x, Y > y).$$

Hence, obtain the marginal p.f.s of X and Y and their joint p.f.

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STATISTICS 230 – Tutorial 7 Problems

T7 – 1. Let X and Y be probabilistically independent random variables with finite means and variances.

(a) Show that $X+Y$ and $X-Y$ are uncorrelated if and only if $var(X) = var(Y)$.

(b) Show that $cov(X, XY) = var(X) \bullet E(Y)$.

T7 – 2. A machine works for a time X_1 until it breaks down; it is then repaired, which takes time Y_1. It then works for a further time X_2 until it breaks down again; the new repair time is Y_2, and so on. The X_js can be modelled as probabilistically independent random variables with common mean μ and common variance σ^2. The value of Y_j depends on how 'old' the machine is, where age is measured in terms of the time it has been working: $X_1 + X_2 + ... + X_j$. It may be assumed that Y_j can be modelled as shown at the right, with $\beta > 0$ and variation negligible compared with σ^2. When the machine breaks down for the n*th* time, it is scrapped immediately. Find the mean and variance of the time between the installation of a new machine and its scrapping.

$$Y_j = \alpha + \beta(X_1 + X_2 + ... + X_j)$$

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STATISTICS 230 – Tutorial 8 Problems

T8 – 1. In a row of $n+1$ points, any point has probability p of being black and probability $q=1-p$ of being white; colour is determined independently for each point. Show that, among the n pairs of adjacent points, the number in which the two points are of *different* colours has mean $2npq$ and variance $2(2n-1)pq - 4(3n-2)p^2q^2$.

HINT: Define indicator variable $X_j = 1$ if points j and $j+1$ have different colours ($j = 1, 2, \ldots n$).

T8 – 2. A multiple choice examination has 100 questions, each with 5 possible answers; one mark is awarded for a correct answer, and one-quarter mark is deducted for an incorrect answer. A particular student has probability p_j of knowing the correct p_i of knowing the correct answer to the jth question, independently of other questions.

(a) Suppose that on questions where he or she does *not* know the answer, they guess randomly among the five possible answers. Show that the student's total mark on the examination has mean Σp_j and variance $\Sigma p_j(1-p_j) + (100 - \Sigma p_j)/4$.

(b) What would be the mean and standard deviation of the student's total mark if he or she *refrained from guessing* when they did not know an answer?

STATISTICS 230 – Tutorial 9 Problems

T9 – 1. My neighbour and I have identical floodlamps whose lifetimes are exponentially distributed with mean $\theta = 300$ hours; each of us burns our floodlamp for six hours per night. Find the probability that:

(a) my floodlamp lasts longer than 60 nights;

(b) both floodlamps last longer than 60 nights;

(c) both floodlamps burn out on the same night.

T9 – 2. Let $X_1, X_2, \ldots\ldots, X_n$ be probabilistically independent random variables each having the p.d.f. given at the right. Let random variable M be the *minimum* of $X_1, X_2, \ldots\ldots, X_n$; find the p.d.f. of M.

$$f(x) = e^{-(x-\beta)} \quad \text{for } x > \beta > 0$$

T9 – 3. According to the Maxwell-Boltzmann Law, the velocity V of a gas molecule is a continuous random variable with p.d.f. as given at the right, where k and β are positive constants. The kinetic energy of a molecule of mass m is $Y \equiv \frac{1}{2}\mathrm{m}V^2$.

$$f(v) = \mathrm{k}v^2e^{-\beta v^2} \quad \text{for } v > 0$$

(a) Show that $\mathrm{k} = 4\beta^{3/2}/\sqrt{\pi} = 4\beta^{3/2}/\Gamma(\frac{1}{2})$.

(b) Find the p.d.f. and expected value of Y.

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STATISTICS 230 – Tutorial 10 Problems

T10 – 1. Suppose that the lifetimes of television picture tubes can be modelled by a normal distribution; an investigation of tubes produced by one manufacturer shows that 15% of tubes fail before 2 years and 5% last longer than 6 years.

(a) Find the mean and standard deviation of the lifetime distribution of these tubes.

(b) Find the probability that the total lifetime of two randomly chosen tubes exceeds 10 years.

T10 – 2. In a club of n voting members, a block of s members votes unanimously on any issue; the other $t = \mathrm{n} - s$ members vote at random ($p = ½$). All n members always vote and t is fairly large. How large must s be (in terms of t) so that there will be an 84.1% chance that the block of s voters will carry any issue for which it votes?

T10 – 3. The distribution of bag weights in large shipments of potatoes can be modelled by a $N(\mu, 4)$ distribution, where μ is unknown. To decide whether to accept a shipment, an inspector is allowed to weigh 10 randomly chosen bags; ideally, she would like to accept only shipments with $\mu \geq 100$ and reject all those with $\mu < 100$. Three acceptance rules for shipments are considered:

I: accept if at least 7 of the ten bags weigh more than 99 pounds;

II: accept if the smallest of the ten bags weighs more than 96.6 pounds;

III: accept if the average of the ten bag weights is more than 99.8 pounds.

For each rule, determine the acceptance probability as a function of μ, and plot the three functions for $98 \leq \mu \leq 102$. Which rule is the best? Explain briefly