UNIVERSITY OF WATERLOO
WATERLOO ONTARIO

STATISTICS 230

MIDTERM EXAMINATION: November 2, 1989

ATTEMPT ALL QUESTIONS; ALL ARE OF EQUAL MARK VALUE

AIDS: Calculator

TIME: 2 hours

INSTRUCTORS: W.H. Cherry (10.30 a.m.)
Chris Springer (2.30 p.m.)
Cyntha Struthers (St. Jeromes)

NAME (Print) _____

Signature _____

I.D. No. _____

INSTRUCTOR _____

QUESTION	MARK
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

INSTRUCTIONS:

1. Before you begin the examination, make sure that you have all 6 pages

2. Answer each question in the space provided under the statement of the question. *If* more space is needed, use the page opposite the one on which the question is presented. Make *sure* that any continuation of a solution is clearly labelled.

3. This complete booklet of question-answer sheets is to be submitted for grading.

4. The *method* of solution of the questions on this examination is important; it may be *more* important than the final answer to the question. An *essential* feature of the method is the clarity of its presentation.

5. To promote efficient grading, most questions (or parts of questions) on this examination have a box or other area in which to indicate your answer. Hence, *after you have completed the solution of a question to the best of your ability*, write the relevant answer or expression (if you have obtained one) in the appropriately labelled area at the right hand side of the page.

MARKS

10

(4, 2, 4)

1. Suppose that there are 2,000 undergraduates in the Faculty of Mathematics and that 700 of them are female; suppose also that 15 mathematics undergraduates attend training sessions for the Putnam Mathematical Competition. Let random variable X be the number of females training for this Competition.

   (a) Give an expression for the probability function, $f(x)$.

   (b) Explain briefly the main assumption underlying the use of the model in (a).

   (c) Find an expression for the probability that $X = 2$ and then calculate its value using the binomial approximation to the expression.

(a)

Probability function (a)

(b)

(c)

Value (c)

MARKS

10

(3, 3, 4)

2. A club has 9 men and 16 women members; they comprise 6 married couples and 13 singles. A committee of 5 people is formed by choosing at random from among all 25 club members. Find the probability that the committee contains:

(a) a married man, a single man, a married woman, and 2 single women;

(b) a married couple, a single man, and 2 single women;

(c) one married couple, given that it contains 2 men.

(a)

(a) Probability

(b)

(b) Probability

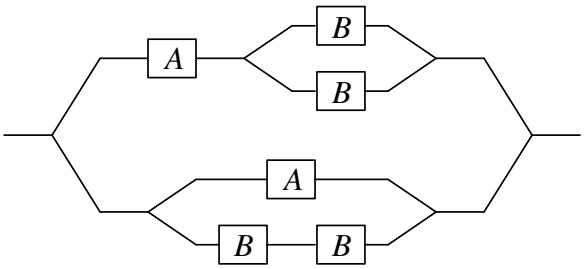
(c)

(c) Probability

MARKS

10

3. An electrical circuit has switches connected as shown at the right. Switches operate independently and the probability a switch is closed is 0.7 for type *A* and 0.6 for type *B*. What is the probability that there is at least one complete path for current flow through the circuit?



Probability

4. Describe three assumptions that are required when using the Poisson distribution as a model for the number of customers arriving at a restaurant during a 10-minute period over the lunch hour.

MARKS

10

(2, 1, 7)

(b) Comment on how closely you expect the three assumptions in (a) to be met in practice.

(c) Assume that it is permissible to use a Poisson model with an average of λ customers arriving per minute at the restaurant. If it is known that n customers arrived during a period of m minutes, show that the number, X, who arrived during the first k minutes of this period has a binomial distribution; your demonstration must include an explicit statement of the relevant binomial probability function.

(a) Ass. #1:

Ass. #2:

Ass. #3:

(b)

(c)

Binomial probability function (c)

5. A debt collection agency keeps records on the number of times delinquent customers have to be contacted before payment of a debt is made; the agency has found that the probabiliy of payment is 0.3 for any contact, and contacts are considered to be probabilistically independent.

MARKS

10

(3, 3, 4))

(a) If random variable X ($X = 1, 2, 3, ...$) is the total number of contacts needed to collect a debt, find the probability function for X; explain your reasoning.

(b) Find the probability that $X \geq 4$.

(c) Give an *expression* for the probability that, when records on 100 random delinquent customers are checked, the results will be as shown at the right.

x	1	2	3	≥4	Total
Frequency	26	20	19	35	100

(a)

Probability function (a)

(b)

Probability (b)

(c)

Expression (c)