UNIVERSITY OF WATERLOO

WATERLOO ONTARIO

STATISTICS 230

FINAL EXAMINATION: August, 1988

ATTEMPT ALL QUESTIONS; NOT ALL ARE OF EQUAL MARK VALUE

AIDS: Formula sheet (8½ x 11 inches, one side, handwritten only)
Table of the normal distribution (loose copy, *not* the text)
Calculator

TIME: 3 hours

INSTRUCTOR: W. H. Cherry

NAME (Print) _____

Signature _____

I.D. No. _____

QUESTION	MARK
1	/ 9
2	/ 9
3	/ 11
4	/ 9
5	/ 12
6	/ 11
7	/ 10
8	/ 9
9	/ 10
Total	/ 90
Total	/ 60
Quiz #1	/ 20
Quiz #2	/ 20
FINAL	/100

INSTRUCTIONS:

1. Before you begin the examination, make sure that you have all 10 pages
2. Answer each question in the space provided under the statement of the question. *If* more space is needed, use the page opposite the one on which the question is presented. Make *sure* that any continuation of an answer is clearly labelled.
3. This complete booklet of question-answer sheets is to be submitted for grading.
4. The *method* of answering the questions on this examination is important; it may be *more* important than the final answer to the question. An *essential* feature of the method is the clarity of its presentation.
5. To promote efficient grading, most questions (or parts of questions) on this examination have a box or other area in which to indicate your answer. Hence, *after you have completed the answer to a question to the best of your ability*, write the relevant answer or expression (if you have obtained one) in the appropriately labelled area at the right hand side of the page.

MARKS

9

(3, 3, 3)

1. In a lottery, each ticket contains six numbers chosen without replacement from 1, 2, ..., 39. The six winning numbers are chosen at random without replacement from 1, 2, ..., 39; the *order* in which these six numbers are drawn is irrelevant.

   (a) To win first prize, a ticket must contain *all* the six numbers drawn; find the probability that a person with one ticket wins first prize.

   (b) To win fifth prize, a ticket must contain exactly three of the six numbers drawn; find the probability that a person with one ticket wins fifth prize.

   (c) If the lottery is conducted weekly, find the probability that at least one of the six numbers drawn this week was also drawn last week.

(a) (a)

Probability

(b) (b)

Probability

(c) (c)

Probability

MARKS

9

(4, 5)

2. Two teams play a best-of-seven series. The first two games are to be played on *A*'s field, the next three games on *B*'s field, and the last two (if needed) on *A*'s field. The probability that *A* wins a game is 0.75 at home and 0.4 away. Find the probability that:

(a) *A* wins the series in 4 games; in 5 games;

(b) the series does *not* go to 6 games.

(a)

(a)

Probability: 4 games

(a)

Probability: 5 games

(b)

(b)

Probability

MARKS

11 (3, 4, 4)

3. An urn contains 5 red balls and 3 black balls. Three balls are drawn one at a time, the contents of the urn being thoroughly mixed before each draw. Tabulate the probability function of random variable X, the number of red balls drawn, in each of the following cases:

(a) after a ball has been drawn it is replaced in the urn before the next drawing;

(b) balls are *not* replaced after being drawn;

(c) after a ball has been drawn, it is replaced by one of the *other* colour before the next drawing.

(a)

x	
$f(x)$	

(a)

(b)

x	
$f(x)$	

(b)

(c)

x	
$f(x)$	

(c)

MARKS

9

(4, 5)

4. A book of 200 pages contains 150 misprints. Assuming that each misprint occurs on a random page, independently of other misprints, find a numerical *expression* for the probability that:

(a) a particular page contains fewer than three misprints;

(b) a five-page section of the book contains more than four misprints.

Use the Poisson approximation to evaluate these two probabilities.

(a)

_____ (a)
Numerical probability expression

[ ] (a)
Probability

(b)

_____ (b)
Numerical probability expression

[ ] (b)
Probability

MARKS

5. Let X be the largest outcome and Y the smallest when three balanced dice are rolled. The table at the right gives $216 \times \Pr(X = x, Y = y)$.

12 (a) Tabulate the marginal probability function of Y and the conditional probability function of X given that $Y = 2$.

(3, 5, 4) (b) Find the mean and variance of X and of Y and their covariance.

(c) Find the mean and variance of the *range*, $R \equiv X - Y$.

	$x=1$	2	3	4	5	6
$y=1$	1	6	12	18	24	30
2	0	1	6	12	18	24
3	0	0	1	6	12	18
4	0	0	0	1	6	12
5	0	0	0	0	1	6
6	0	0	0	0	0	1

(a)

y	
$f(y)$	

x	
$f(x\|y=2)$	

(b)

		(b)
$E(X)$	$var(X)$	
		(b)
$E(Y)$	$var(Y)$	
		(b)
	$cov(X, Y)$	

(c)

		(c)
$E(R)$	$var(R)$	

MARKS

11

(5, 2, 2, 2)

6. A continuous random variable Y has the probability density function given at the right.

$$f(y) = \begin{cases} ky^{-4} & ; \quad y \geq 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(a) Evaluate the constant k and find the c.d.f. of Y; sketch the p.d.f. of Y.

(b) Evaluate: $\Pr(-2 \leq Y \leq 3)$.

(c) Find the mean of Y.

(d) Derive the p.d.f. of random variable $Z = \log_e Y$.

(a)



(a)

k



(a)

C. d. f. of Y



(a)

(b)



(b)

Probability

(c)



(c)

$E(Y)$

(d)



(d)

P. d. f. of Z

MARKS

10

(4, 2, 2, 2)

7. Suppose that the lifetime, T, of a particular type of light bulb can be modelled by an exponential distribution with mean θ days. The lifetimes (in days) observed for a random sample of 30 bulbs are shown at the right. The average of these 30 lifetimes, $\bar{t} = 60$ days, can be taken as an estimate of the parameter θ. Use this exponential model as the basis of the following calculations.

24	262	88	8	121	15
63	48	226	72	248	21
42	20	5	12	120	11
3	14	71	11	14	11
16	90	1	16	52	95

(a) Evaluate: $\Pr(0 \le T \le 40)$; $\Pr(40 < T \le 100)$; $\Pr(100 < T \le 200)$; $\Pr(T > 200)$.
Compare the expected frequencies with the observed frequencies in these four intervals; comment briefly.

(b) Find the probability that a new bulb, selected at random, will last longer than 40 days.

(c) If a bulb has already lasted for 40 days, find the probability that it will last at least another 40 days.

(d) Find the expected total lifetime of a component which has already lasted for 40 days.

(a)

$\Pr(0 \le T \le 40)$	$\Pr(40 < T \le 100)$	$\Pr(100 < T \le 200)$	$\Pr(T > 200)$	
				(a)

(b)

Probability	
	(b)

(c)

Probability	
	(c)

(d)

Total lifetime	
	(d)

MARKS

9

(3, 3, 3)

8. The examination scores obtained by a large group of students can be modelled by a normal distribution with a mean of 65% and a standard deviation of 10%. Using this model:

(a) find the percentage of students who obtain each of the following letter grades:

A (≥80%); *B* (70-80%); *C* (60-70%); *D* (50-60%); *F* (<50%);

(b) find the probability that the average score of a randomly-chosen group of 100 students is greater than 67%;

(c) find the probability that the average scores of two distinct randomly-chosen groups of 25 students differ by more than 5 marks.

(a)

%	%	%	%	% (a)
A	*B*	*C*	*D*	*F*

(b)

Probability (b)

(c)

Probability (c)

MARKS

9. Customers at a fast food franchise may choose to have 0, 1, 2 or 3 slices of dill pickle on their hamburgers; experience has shown that the proportions of customers making the four choices are 20%, 40%, 30% and 10% respectively, and that 400 hamburgers are sold during the lunch hour. Also, a dill pickle makes 5, 6 or 7 slices with probabilities of 0.4, 0.5 and 0.1 respectively. Find the approximate probability that 100 dill pickles will make enough slices to cover the lunch hour business.

10

Probability