UNIVERSITY OF WATERLOO

WATERLOO ONTARIO

STATISTICS 230

FINAL EXAMINATION: December, 1989

ATTEMPT ALL QUESTIONS; MARK VALUES VARY AMONG QUESTIONS

AIDS: Formula sheet (attached)
Table of the normal distribution (attached)
Calculator

TIME: 3 hours

INSTRUCTORS: W. H. Cherry (10.30 a.m.)
Chris Springer (2.30 p.m.)
Cynthia Struthers (St. Jeromes)

NAME (Print) _____

Signature _____

I.D. No. _____

INSTRUCTOR _____

TUTORIAL SECTION _____
(Sections C01-C08 only)

QUESTION	MARK
1	/ 7
2	/ 9
3	/ 10
4	/ 14
5	/ 13
6	/ 16
7	/ 12
8	/ 9
9	/ 10
Total	/100
Total	/ 70
Assignments	/ 10
Midterm	/ 20
FINAL	/100

INSTRUCTIONS:

1. Before you begin the examination, make sure that you have all 10 pages
2. Answer each question in the space provided under the statement of the question. *If* more space is needed, use the page opposite the one on which the question is presented. Make *sure* that any continuation of an answer is clearly labelled.
3. This complete booklet of question-answer sheets is to be submitted for grading.
4. The *method* of answering the questions on this examination is important; it may be *more* important than the final answer to the question. An *essential* feature of the method is the clarity of its presentation.
5. To promote efficient grading, most questions (or parts of questions) on this examination have a box or other area in which to indicate your answer. Hence, *after you have completed the answer to a question to the best of your ability*, write the relevant answer or expression (if you have obtained one) in the appropriately labelled area at the right hand side of the page.

MARKS

7

1. Three people, Joe, Sue and Tom, write an exam; Joe has a 90% chance of passing, Sue an 80% chance and Tom a 20% chance. Find the probability that exactly two of the three people pass the exam.

Probability

MARKS

9

2. Suppose that 5% of Canadian households do not have a colour TV set, 55% have one set and 40% have 2 or more sets. Only 2% of the households without a colour TV have a VCR, while 30% of households with one colour TV and 70% of those with 2 or more colour TVs have a VCR. If a randomly-chosen Canadian household has a VCR, what is the probability it has 2 or more colour TV's?

Probability

MARKS

10

(5, 5)

3. The number of accidents at a particular intersection can be modelled by a Poisson distribution with an average of 1.2 accidents per month.

(a) Find the probability that there will be 5 accidents at the intersection in a 3-month period.

(b) Find the probability that the time between consecutive accidents is between ¼ and ½ a month.

(a)

Probability (a)

(b)

Probability (b)

MARKS

14

(2, 4, 6, 2)

4. The joint probability function of random variables X and Y is as shown in the table at the right. Find:
   (a) $E(X)$ and $E(Y)$;
   (b) $var(X)$ and $var(Y)$;
   (c) $cov(X, Y)$ and $var(2X - Y)$.
   (d) Are X and Y probabilistically independent? Explain briefly.

$f(x,y)$	$x=0$	1	2	
$y=0$	0.01	0.03	0.24	
1	0.02	0.05	0.11	
2	0.27	0.17	0.10	

(a)

(a) $E(X)$

(a) $E(Y)$

(b)

(b) $var(X)$

(b) $var(Y)$

(c)

(c) $cov(X, Y)$

(c) $var(2X - Y)$

(d)

(d) X, Y independent?

MARKS

13

(2, 5, 6)

5. The fraction, X, of a 3,000 km auto race which a car completes before dropping out has the probability density function given at the right.

$$f(x) = \begin{cases} kx^2 & ; \quad 0 < x \le 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(a) Evaluate the constant k.

(b) Find the mean and variance of X.

(c) The driver gets paid (in dollars) $Y \equiv X^2$. Find the p.d.f. of Y.

(a)

k (a)

(b)

$E(X)$ (b)

$var(X)$ (b)

(c)

P.d.f. of Y (c)

6. The viscosity of the oil in cans of S.A.E. 30 motor oil has a normal distribution with mean 30.0 and variance 1.85 (in appropriate uni). In each of the following cases, find the probability that:

MARKS

16

(3, 4, 4, 5)

(a) the viscosity of the oil in a randomly-chosen can lies between 29.2 and 30.5 units;

(b) the average viscosity of the oil in 5 randomly-chosen cans is more than 30.1 units;

(c) the viscosiy of the oil in 2 (independent) randomly-chosen cans differs by more than 0.5 units;

(d) for a random sample of 60 independent cans, exactly 17 have viscosities under 29.2 units and exactly 24 have viscosities above 30.5 units. [Give **an expression only** for this probability.]

(a)

(a) Probability

(b)

(b) Probability

(c)

(c) Probability

(d)

(d) Probability expression

MARKS

12

(4, 8)

7. A company assembling stereo components has a supply of 700 nuts and 500 washers; each component assembled requires one nut and one washer. If 80 pieces are chosen at random from the box which contains the nuts and the washers mixed together:

(a) give an *expression* for the exact probabiliy that at least 25 components can be assembled from the 80 pieces;

(b) find an approximate value of the probability that at least 25 components can be assembled from the 80 pieces.

(a)

(a)

Probability expression

(b)

(b)

Probability

MARKS

8. Let X and Y be independent identically distributed random variables with probability function as shown at the right.

$$f(x) = p(1-p)^x \quad \text{for } x = 0, 1, 2, ...$$

9 (a) Show that $T \equiv X+Y$ has a negative binomial distribution.

(6, 3) (b) Show that the conditional distribution of X, given that $X+Y=t$, is uniform on 0, 1, ..., t.

(a)

(a)

P. f. of $T \equiv X+Y$

(b)

(b)

P. f. of X given $X+Y=t$

MARKS

10

(6, 4)

9. A multiple choice examination has 100 questions, each with 5 possible answers; one mark is awarded for a correct answer, and one-quarter mark is deducted for an incorrect answer. A particular student has probability p_j of knowing the correct answer to the jth question, independently of other questions.

   (a) Suppose that on questions where he or she does *not* know the answer, they guess randomly among the five possible answers. Show that the student's total mark on the examination has mean Σp_j and variance $\Sigma p_j(1-p_j) + (100-\Sigma p_j)/4$.

   (b) Show that the total mark on the examination for a student who *refrains from guessing* when he or she does not know an answer has mean Σp_j [*i.e.*, as in (a)] but variance $\Sigma p_j(1-p_j)$.

(a)

(b)