University of Waterloo — STAT 221 – W. H. Cherry

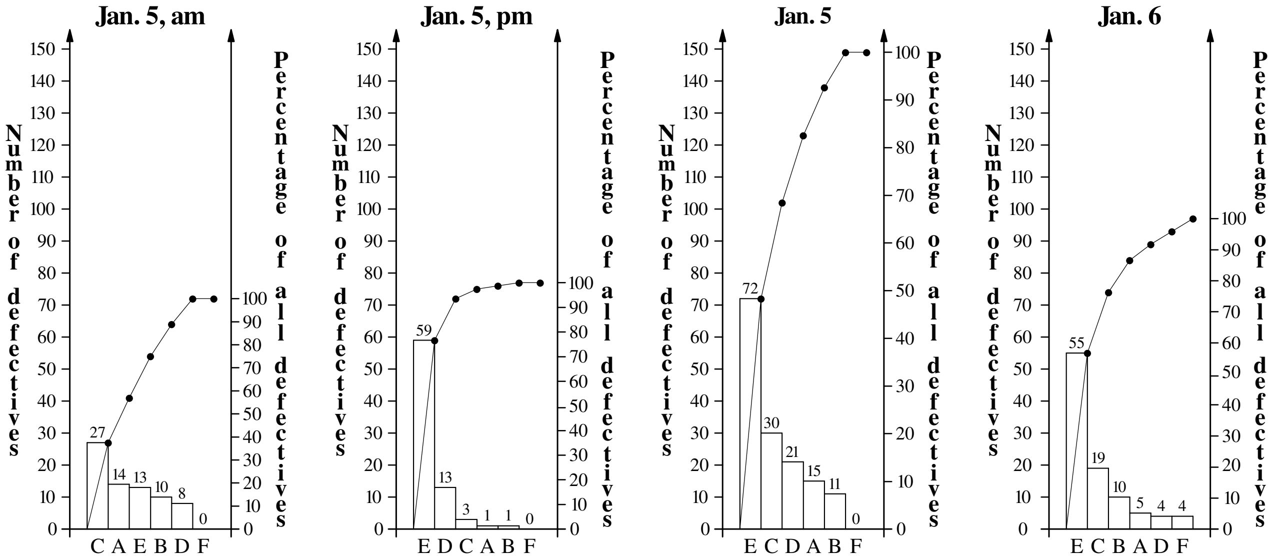
Assignment 3 Outline Solution

A3 – 1. We can tabulate the number and percentage of each type of defect in the relevant time periods as follows:

Defect Type	Jan. 5, am No.	Jan. 5, am %	Jan. 5, pm No.	Jan. 5, pm %	Jan. 5 No.	Jan. 5 %	Jan. 6 No.	Jan. 6 %
A	14	19	1	1	15	10	5	5
B	10	14	1	1	11	7	10	10
C	27	38	3	4	30	20	19	20
D	8	11	13	17	21	14	4	4
E	13	18	59	77	72	48	55	57
F	0	--	0	--	0	--	4	4
	72	100	77	100	149	99	97	100

A unfill on outer surface
B unfill on inner surface
C visual defect on outer surface
D colour out of specification
E surface scratch
F other

(a) Pareto diagrams (with defect *numbers* given at the tops of the bars) are then:



(b) Comparing the third and fourth Pareto diagrams, we see that the percentages of each type of defect are roughly comparable over the two days – E is *most* common and represents a somewhat higher percentage of all defects on January 6 than on January 5; F is *least* common and occurs *only* on January 6. Defects A and D are also perhaps somewhat higher on January 5.

(c) Comparing the first and second diagrams, we see that there is a marked difference in the distribution of defects after the regular operator was replaced at around noon on January 5 – the predominant defect became E (the surface scratch) and three defects important in the morning (*viz*. A, B and C) became *much* less so. Defect E persisted on January 6 when the regular operator returned, and so does *not* seem to be an operator effect; the production set-up should be examined to try to see if something has changed recently to start making the surface scratches noticably more frequent. There is some evidence of an operator effect in defects A, B and C, which became appreciably more common again on January 6 when the regular operator returned, compared with their marked *de*crease with the replacement operator.

A3 – 2. Each group or individual will have their own way of approaching this question, although there should be significant elements in common among the finished diagrams; compare your diagram with that of another group or individual, and discuss the similarities and differences in the context of how each diagram was arrived at.

A3 – 3. (a) The best time (from a sampling perspective) to check the braces is as they are being unpacked. It would be inconvenient in a manufacturing environment to number the braces 1 to N (either physically or conceptually) and then take for inspection those braces corresponding to 200 random numbers in the interval 1 to N. Instead, one could inspect every kth brace as they are unpacked, with $200k = N$. A possible *dis*advantage of this *systematic selecting* procedure is the requirement to unpack *all* the braces, which is inconvenient if it turns out that they have to be returned to the supplier.

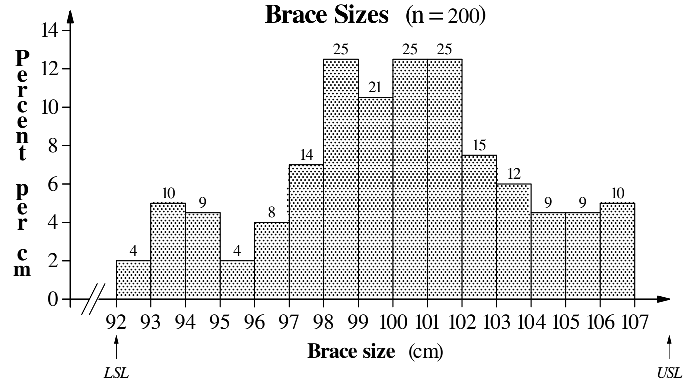
(*continued overleaf*)

Assignment 3 Outline Solution (continued 1)

A3 – 3. (cont.) (b) The numbers and percentages of observations in 15 intervals are:

	[92, 93)	[93, 94)	[94, 95)	[95, 96)	[96, 97)	[97, 98)	[98, 99)	[99, 100)	[100, 101)	[101, 102)	[102,103)	[103, 104)	[104, 105)	[105, 106)	[106, 107)
No.	4	10	9	4	8	14	25	21	25	25	15	12	9	9	10
%	2	5	4½	2	4	7	12½	10½	12½	12½	7½	6	4½	4½	5

(c) The histogram (shown at the right) raises serious questions about the manufacturing process for the braces. There appears to be too much variation – it is possible that we are looking at more than one process stream. In addition, the cut-off on the right-hand side is rather abrupt; a more normal distribution of sizes would be expected. This may indicate that extreme non-conforming product at the high end has been inspected out, which would usually be a sign of a poorly managed manufacturing process.



A3 – 4. To cover 99.7% of the production, we need the average ± 3 standard devations;
this interval is: $0.239 \pm 3 \times 0.003 = (0.230,\ 0.248)$ cm.
A key *assumption* here is that the rotor shaft diameters can be modelled by a *normal* distribution.

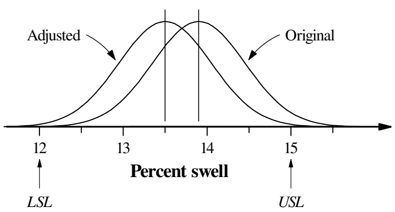
A3 – 5. (a) Let the random variable S represent the amount of swell (in percent) of a plastic rod selected equiprobably from the rods made by the extruder;
we use the model: $S \sim N(13.9,\ 0.55)$,
where the values for the mean and standard deviation of the normal model are those of the average and standard deviation of the swell observed for the rods produced by the process.

Then: proportion out of specification $= 1 - \Pr[12.0 \le N(13.9,\ 0.55) \le 15.0]$
$= 1 - \Pr[-3.\dot{4}\dot{5} \le N(0,\ 1) \le 2]$
$= 0.023\,025\,743$;

thus, for 10,000 rods, we expect about 230.3 or around 230 to be out of specification.

(b) *Assuming* that the standard deviation of the swell of the rods from the extrusion process remains *un*changed by the adjustment of the process average, and also that the normal model is still applicable, we have:

proportion out of specification
$= 1 - \Pr[12.0 \le N(13.5,\ 0.55) \le 15.0]$
$= 1 - \Pr[-2.\dot{7}\dot{2} \le N(0,\ 1) \le 2.\dot{7}\dot{2}]$
$= 2 \times 0.003\,193 = 0.006\,386 \simeq 0.64\%$.



We see that *centering* the process in the tolerance interval has reduced the proportion of rods which do not meet specifications by nearly a factor of 4 (from about 2.3% to about 0.64%). In the diagram above at the right, these proportions are represented by the tail areas below 12 and above 15 under the appropriate normal curve.

(c) For the adjusted process: $\Pr[N(13.5,\ 0.55) \ge 15.4] = \Pr[N(0,\ 1) \ge 3.\dot{4}\dot{5}] = 0.000\,275\,611$;

i.e., this calculation shows that if the process average is *really* 13.5%, then under the normal model there is a probability of only about 1 in 3,600 of observing, by chance, a swell reading of 15.4% or higher; this probability is so low that a more likely explanation than mere chance is that the process average is *not* 13.5%, but is actually *greater* than this value.