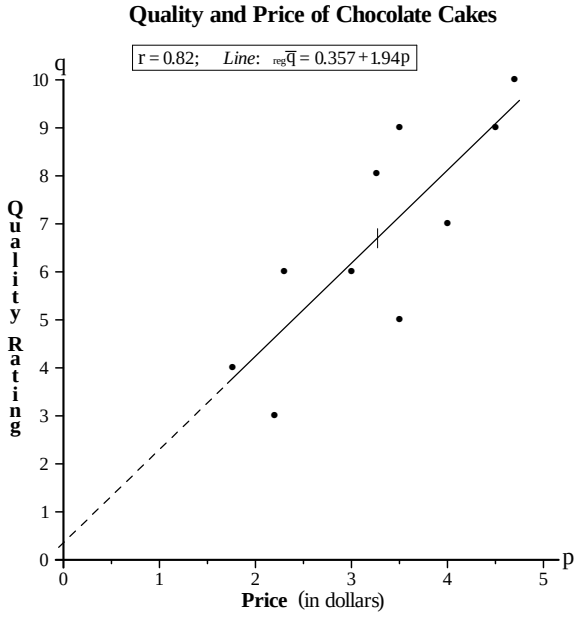
Figure 13.6. SIMPLE LINEAR REGRESSION: Correlation

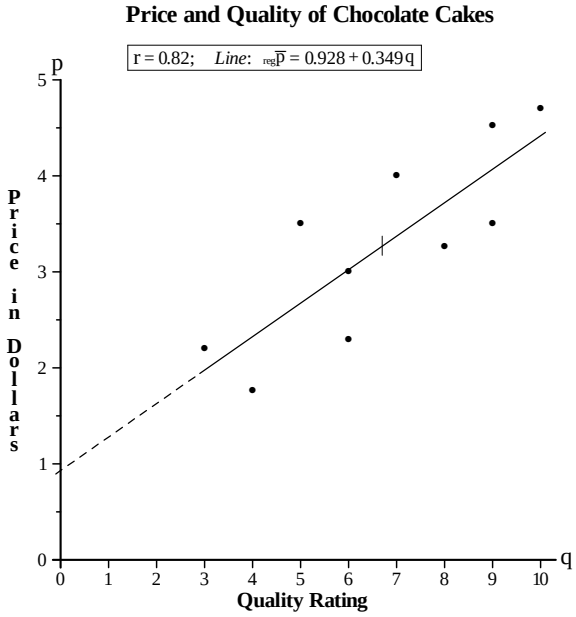
Program 9 in: *Against All Odds: Inside Statistics*

This program contains two main segments. The first introduces the *correlation coefficient* r, an important measure of the strength and direction of the straight-line association between two *quantitative* variates. The video illustrates the use of correlation in describing the results of a study of identical (or monozygotic) twins who have been raised apart. These twins have a strong correlation between physical characteristics and, more surprisingly, a moderately strong correlation between mental and behavioural characteristics. Correlation always satisfies the inequality $-1 \le r \le 1$, and $r = \pm 1$ only in the case of *perfect* linear association. The value of r is *not* affected by changes in the unit of measurement of either variate.

Linear regression signifies a straight-line relationship between an explanatory variate and a response variate. Correlation applies to *any* two quantitative variates; it does *not* require that one be an explanatory variate. When we have an explanatory variate and a reponse variable, however, correlation and regression are closely related. The second part of this Program describes the relationship between correlation and regression. The *squared correlation coefficient* r^2 (also called the *coefficient of determination*) is the fraction of the variation in one variate accounted for (or 'explained' by) least squares regression on the other variate. The estimated regression of $\mathbf{Y}$ on $\mathbf{X}$ is the straight line with slope $b_1 = rs_y/s_x$ and passing through the point $(\overline{x}, \overline{y})$, the *point of averages*. In the video, the Coleman Report uses r^2 to describe how characteristics of schools are surprisingly poor predictors of student achievement.

The results of computing a correlation or regression must be interpreted with due attention to the following: the possible effects of lurking variates, the lack of resistance of these procedures, the danger of extrapolation, the fact that correlations based on averages (*e.g.*, ecological correlations) are usually *higher* than those for the corresponding individuals, and an understanding that correlation and linear regression measure only *straight-line* relationships to the possible exclusion of *other* important aspects of the data.





Values quoted in the video for some other correlations are:

- Between the heights of identical twins raised *apart* 0.92
- Between personality inventories of identical twins raised *apart* 0.49
- Between personality inventories of identical twins raised *together* 0.52
- Between home run output and strikeouts (in baseball) 0.7

1 From the perspective of a data-based investigating, outline the *weakness(es)* of the data collection aspects, as they are portrayed in the video, of Dr. Amabile's investigation of quality and price of chocolate cakes; give your discussion in point form.

- Indicate briefly how you would *overcome* the weakness(es) you identify.

(*continued overleaf*)

2 What is the implication of the *dashed* parts of the estimated regression lines in the two diagrams overleaf on page 13.37? Explain briefly.

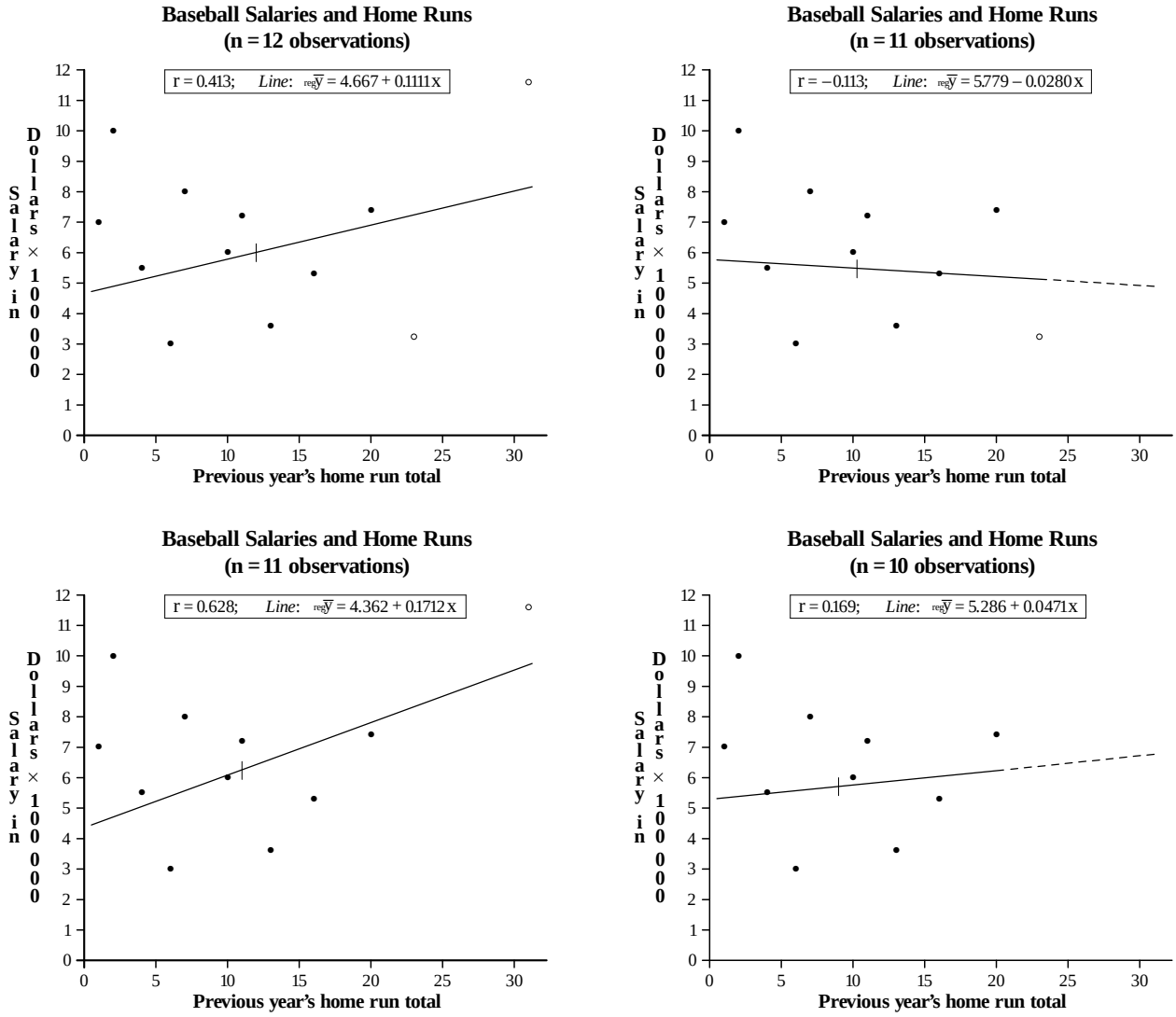
The following data (indexed by $j = 1, 2, \ldots, 12$) are similar to those shown as a scatter diagram in Program 9 for baseball players' salaries *vs* their previous year's home run total:

Player number (j)	1	2	3	4	5	6	7	8	9	10	11	12
Home run total (x_j)	1	2	4	6	7	10	11	13	16	20	23	31
Salary in $\$\times 10^5$ (y_j)	7.0	4.1	5.5	3.0	8.0	6.0	7.2	3.6	5.3	7.4	3.3	11.6;

for these $n = 12$ observations:

$$\sum_{j=1}^{n} x_j = 144, \qquad \sum_{j=1}^{n} x_j^2 = 2{,}642; \qquad \sum_{j=1}^{n} y_j = 72, \qquad \sum_{j=1}^{n} y_j^2 = 498.16; \qquad \sum_{j=1}^{n} x_j y_j = 965.5.$$

The scatter diagram, correlation, and estimated least squares regression line, based on these data, are shown below; all 12 observations are included for the first diagram, and then either or both of the (anomalous) *12*th and *11*th observations are *ex*cluded for the other three plots. The short vertical line crossing each estimated regression line indicates the point of averages $(\bar{x}, \bar{y})$.



3 During the interview shown in the video, Daniel Seligman of *Fortune Magazine* says: *The players .... are going up there trying to hit that long ball and make big bucks.* Comment briefly on this statement in light of the information presented above.

4 What matter(s) of general relevance to data analysis are illustrated by the information presented above? Explain briefly for each matter you describe.