Assignment 5

A5 – 1. The Rockwell hardness index for steel is determined by pressing a diamond point into the steel with a specified force and measuring the depth of penetration. For a certain type of steel, a manufacturer claims that the true average hardness of this type of steel made by his process is *at least* 64 units. For 50 specimens of the steel, obtained by judgement selecting by an experienced supervisor at the manufacturing site, the hardness was found to average 62 units. Use a test of *hypothesis* with $\alpha = 0.05$ to decide whether to accept the manufacturer's claim, assuming that the Rockwell hardness is *known* from experience to have a standard deviation of 8 (in suitable units).

A5 – 2. An important quality characteristic of automobile batteries is their weight, because it is sensitive to the amount of lead in the battery plates. A particular brand of heavy-duty battery has a weight specification of 30 kg; it is known from experience that the standard deviation of the weight is 0.55 kg. Forty of these batteries selected equiprobably from a large production run have an average weight of 29.6 kg. Use a test of *hypothesis* with $\alpha = 0.01$ to decide if the manufacturing process for the batteries is significantly *below* target with respect to battery weight during the large production run.

A5 – 3. A new prescription drug to be marketed in tablet form is supposed to contain 5 mg of codeine per tablet. Health and Welfare Canada (HWC) must decide whether to allow the sale of this medication. Denoting the true average weight (in mg) of codeine per tablet as $\overline{\mathbf{Y}}$, represented by the model mean μ, HWC decides to carry out a test of the hypothesis H_o: $\mu = 5$ versus the hypothesis H_a: $\mu \neq 5$ as part of their investigation.

(a) Explain briefly why the above alternative hypothesis was chosen rather than H_a: $\mu > 5$ or H_a: $\mu < 5$.

(b) For the null and alternative hypotheses chosen by HWC, describe the type I and type II errors, and the consequences of making them, in the context of this investigation.

A5 – 4. Warning flares of the type contained in most automobile emergency kits are sometimes defective in that they fail to ignite. A consumer advocacy group is to investigate a claim that the proportion of flares that fail to ignite is much higher than the value of 0.1 given by the manufacturer. A large sample of the flares, selected equiprobably, will be tested and the results used to decide between the hypothesis H_o: $\pi = 0.1$ and the hypothesis H_a: $\pi > 0.1$, where the model parameter π represents $\mathbf{P}$, the true proportion of defectives for the population of all flares made by this manufacturer; if H_o is rejected, a charge of false advertising will be filed against the manufacturer.

(a) Explain briefly why the above alternative hypothesis was chosen rather than H_a: $\pi \neq 0.1$ or H_a: $\pi < 0.1$.

(b) For the null and alternative hypotheses chosen by the advocacy group, describe the type I and type II errors, and the consequences of making them, in the context of this investigation.

A5 – 5. Toxicologists have found that the wet weight (in mg, and represented by random variable W, say) of individual white sucker larvae one day posthatch can be modelled by a normal distribution with a mean of 6.9 mg and a standard deviation of 2.6 mg.

(a) Find the probability that an equiprobably-selected larva has a wet weight that is greater than 8.5 mg.

(b) Find the probability that the *average* wet weight of a random sample of 25 larvae is greater than 8.5 mg.

(c) Explain briefly whether an average wet weight that just exceeds 8.5 mg for a random sample of 25 larvae provides good evidence that the larvae are *older* than one day; show your reasoning clearly.

(d) Repeat (c) but now set out the reasoning *formally* as a test of *significance*.

A5 – 6. A quality engineer in an automobile engine plant measures a critical dimension (in mm) of a sample of crankshafts at regular intervals; the latest 16 measurements are as follows:

224.120	224.001	224.017	223.982	223.989	223.961	223.960	224.089
223.987	223.976	223.902	223.980	224.098	224.057	223.913	223.999;

numerical summaries of these n = 16 observations are:

sum of observations: 3,584.031; sum of squared observations: 802,829.945 349.

The target for the dimension is 224 mm, and it is known from experience that its standard deviation is 0.060 mm.

(a) Use an appropriate test of significance to assess the evidence provided by these data about whether the process for manufacturing the crankshafts is centred on its target.

(b) Find an approximate 90% confidence interval for the true average dimension of the crankshafts produced at the engine plant.

(*continued overleaf*)

***A5 – 7.** An electricity generating station discharges its cooling water into a river; an environmental impact assessment has determined that, as long as the average temperature of the cooling water does not exceed 65°C, there will be no adverse effects on the river's ecosystem. To check whether the station is meeting this requirement, the temperature is measured for each of 50 samples of cooling water taken at the point of discharge at 1-hour intervals over 2 days. The discrepancy measure z (given at the right) will be used to decide between the hypotheses H_o: $\mu = 65$ and H_a: $\mu > 65$, where μ is the model parameter representing the true average temperature (in °C) of the cooling water discharged by the generating station. (Assume that $\sigma = 5$°C.)

$$z = \frac{\overline{y} - 65}{\sigma/\sqrt{n}}$$

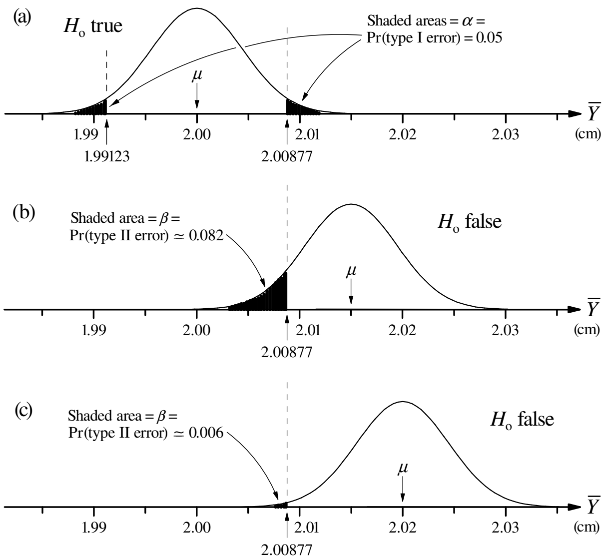
(a) Find the value of α (the probability of making a type I error) corresponding to the rejection of H_o when $z > 1.8$.

(b) If $\mu = 66.5$°C, find the value of β (the probability of making a type II error) corresponding to the rejection region $z > 1.8$ and the hypotheses H_o: $\mu = 65$ and H_a: $\mu > 65$.

(c) If $\mu = 70$°C, repeat the calculation in (b).

(d) Use your answers to (a), (b) and (c) to construct a diagram *like* the one at the right, involving the sampling distribution of $\overline{Y}$. [This diagram is based on the one on p. 491 of the Text, illustrating the discussion of bearing diameters in Example 6.20 – the diagram in (c) shows the additional case $\mu = 2.02$ cm.]

(e) Give an appropriate Answer from the significance test based on the rejection region $z > 1.8$ if $\overline{y} = 66.2$°C; explain briefly the type of error that might have been made in giving this Answer.



A5 – 8. Elevated blood cholesterol levels and being male are major risk factors for heart disease; thus, males with cholesterol levels above 265 mg/dl, who form a not insignificant component of the population, are at appreciably increased risk of having a heart attack. {The average blood cholesterol level for *all* adults in a country like Canada is about 210 mg/dl [*Science 84*, page 16 (April, 1984)].}

A sample selected equiprobably of 100 men with cholesterol levels over 265 mg/dl was treated with the cholesterol-lowering drug cholestyramine. After a suitable period of treatment, the average cholesterol level for these men was found to be 228 with a standard deviation of the *average* (or 'standard error') of 12 mg/dl.

(a) Use an appropriate test of significance to assess the evidence provided by these data about whether the treatment with cholestyramine reduces the average blood cholersterol level of all men above 265 mg/dl to a point where it no longer differs significantly from the national average (210 mg/dl).

(b) Find an approximate 95% confidence interval for the true average blood cholesterol level of men with levels above 265 mg/dl who are treated with cholestyramine.

A5 – 9. As part of an investigation entitled *Effects of Brewery Effluent on Agricultural Soil and Plant Crops* [*Environmental Pollution (Series A)* **33**: 341-351 (1984)], five samples, spread out over the whole area of highly alkaline soil, were treated with a liquid consisting of 25% water and 75% acidic brewery effluent; the average and standard deviation for the pH of the 5 samples were 8.00 and 0.60 respectively.

(a) If the natural pH of the soil from which the 5 samples were taken was 8.75, use an appropriate test of significance to assess the strength of evidence provided by these data about whether the treatment with the effluent mixture really reduced the pH of the highly alkaline soil.

(b) Find an approximate 99% confidence interval for the actual pH of the treated soil.

A5 – 10. Text Exercise 7.33 (page 532): *The design of controls and instruments has a large effect .....*

Numerical summaries of the data given in this question are:

	Right thread	Left thread	Left - Right
Sum of observations	2,603	2,936	333
Sum of squared observations	277,013	362,642	17,061

A5 – 11. Text Exercise 7.38 (page 534): *The following situations all require inference about a mean or means. .....*

 *Optional problem for enrichment