University of Waterloo STAT 221 – W. H. Cherry

Assignment 2

A2 – 1. In the past, half the items received by a retailer from a certain supplier have been in good working order (G), a third have been defective but easily repairable (R), and the rest have been so badly defective they are either impossible or uneconomical to repair (U). If the retailer orders six more items from the same supplier, find the probability:

(a) three items will be G, two will be R and one will be U;
(b) there will be at least one R and exactly one U item;
(c) there will be at least one R item;
(d) there will be at least one item which is *either* R or U.

A2 – 2. Referring to Assignment 1 Question **A1 – 4** about the blackflies, suppose a campground contains 20 tents each with volume 0.6 cubic metres.

(a) Find the probability 6 of the tents contain no blackflies, 4 contain one or two, 3 contain three or four or five, and 6 contain ten or more.
(b) If 6 of the tents contain no blackflies, find the probability 4 contain one or two, 3 contain three or four or five, and 6 contain ten or more.

HINT: Recall the definition of $\Pr(A|B)$.

A2 – 3. Suppose the probabilities of various numbers of mistakes on a page of typing are as shown in the table at the right.

Number of mistakes	0	1	2	3	4	5	6
Probability	0.05	0.25	0.31	0.22	0.10	0.05	0.02

(a) Find the mean and standard deviation of the number of typing mistakes on a page.
(b) Find the approximate probability a 200-page typescript contains at least 500 mistakes.

A2 – 4. Suppose telephone calls arrive at an exchange at an average rate of 3.75 per minute. Find the approximate probability:

(a) *at least* 250 calls arrive at the exchange in a 1-hour period;
(b) *exactly* 250 calls arrive at the exchange in a 1-hour period.

A2 – 5. The probability distribution for the number of people per car crossing a toll bridge is as shown at the right. Would more money be raised by charging (i) \$1.00 per car; or (ii) 40¢ per person; or (iii) 75¢ for the driver and 15¢ per passenger? Show your reasoning clearly.

Number of people	1	2	3	4	5	6
Probability	0.05	0.43	0.27	0.12	0.09	0.04

A2 – 6. Suppose n people take a blood test, and each person has probability π of being infected with a disease, independently of the other people being tested. To save time and money, blood samples from k people are pooled and analyzed together. If this one analysis gives a negative result, it shows that *none* of the k people is infected and the one test suffices for all k people. If the analysis of the pooled blood sample gives a *positive* result, at least one of the k people is infected and they must be tested separately; in this case, a *total* of $k+1$ analyses are necessary for the k people.

(a) Show that, on average, the number of tests required for a group of k people is $k+1-k(1-\pi)^k$.
(b) What is the expected number of tests required for the n/k groups of k people?
(c) Show that, if π is small, the expected number of tests in (b) is approximately $n(k\pi + k^{-1})$, and that it is minimized for $k \simeq 1/\sqrt{\pi}$.

A2 – 7. The 10,000 tickets for a lottery are numbered 0000 to 9999. A four-digit winning number is drawn and a prize is paid on each ticket whose four-digit number is any *arrangement* of the number drawn. For instance, if number 0011 is drawn, prizes are paid on tickets numbered 0011, 0101, 0110, 1001, 1010 and 1100. A ticket costs \$1 and each prize is \$500. The *profit* of the lottery's operator is $10{,}000 - 500Y$, where Y is the number of prizes awarded. Tabulate the probability function of Y, then find the expected value of Y and, hence, determine the operator's expected profit.

A2 – 8. Hamburgers at a fast food outlet can be ordered with 0, 1, 2 or 3 slices of pickle; of all the hamburgers sold, experience has shown that 20% have no pickle, and 40%, 30% and 10%, respectively, have 1, 2 and 3 slices. If 400 hamburgers are to be sold during one supper hour, how many slices of pickle should be on hand to be 99% sure of meeting the demand? State clearly any assumptions needed as part of your solution.

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*A2 – 9. A secretary makes an average of two typing mistakes per page; pages with *more* than two mistakes must be retyped. How many pages can the secretary expect to type so that all 100 pages of a typed report will be acceptable?

*Optional more challenging problem for enrichment