Assignment 1

A1 – 1. A psychiatrist has read in a medical journal of a new treatment which is reported to produce an instantaneous cure in 10% of schizophrenics upon whom it is tried. The psychiatrist decides to check the accuracy of this information by trying the treatment on some equiprobably-selected schizophrenic patients.

(a) Find the smallest number of patients the psychiatrist must plan to try the treatment on so there is a probability of at least 0.5 that at least one patient will be cured instantaneously, if the information in the medical journal about the treatment is correct.

(b) If the information *is* correct, find the probability the first instantaneous cure will occur on the *seventh* schizophrenic patient being treated.

(c) Suppose the psychiatrist decides she must have at least *two* instantaneous cures to be convinced about the efficacy of the treatment. If the information in the medical journal is correct, find the probability she will have to try the treatment on *exactly* ten schizophrenic patients to obtain two instantaneous cures.

A1 – 2. A company recruiter is interviewing applicants for five identical jobs; the probability any one applicant will be accepted is $\frac{2}{3}$. Find the probability:

(a) exactly eight applicants will have to be interviewed to fill all five positions;

(b) eight or fewer applicants will have to be interviewed to fill all five positions.

A1 – 3. A telephone switchboard handles 600 calls an hour on average; the board can make a *maximum* of 20 connections per minute.

(a) Use the Poisson distribution to evaluate the probability the switchboard will be *over*taxed during a given minute.

(b) Find the probability 10 seconds elapse with no calls.

A1 – 4. Suppose blackflies are distributed equiprobably throughout the air in a campground with an average concentration of 15 per cubic metre.

(a) Find the probability of getting exactly three blackflies in a tent of volume 1.2 cubic metres.

(b) Find the probability of getting at least one blackfly in a box of volume 0.04 cubic metres.

(c) For a campsite with three tents each of volume 1.2 cubic metres, find the probability:

(i) there are a total of 40 blackflies in the three tents;

(ii) exactly two of the tents contain exactly three blackflies each.

A1 – 5. Tabulate and graph (as histograms) the values of the function(s) indicated in square brackets [ ] for the following random variables.

(a) A **hypergeometric** random variable (R) with $\mathbf{N} = 10$, $\mathbf{R} = 6$, and $n = 6$ [$f(r)$];
$\mathbf{N}$ is the *total* number of balls in the urn and $\mathbf{R}$ is the number of these balls that are red.

(b) **Binomial** random variables (W, Y, Z) with $n = 10$ and $\pi = 0.2$, 0.5 and 0.9 respectively [$f(w)$, $f(y)$, $f(z)$];
how is the most probable value of each random variable related to the parameters, n and π, of its distribution?

(c) A **negative binomial** random variable (V) with $r = 2$ and $\pi = 0.4$ [$f(v)$];
this notation refers to v failures before the rth success, with Pr(success) = π.

(d) **Poisson** random variables (S, T) with $\mu = 1$ and 10 respectively [$f(s)$, $f(t)$];
how is the most probable value of each random variable related to the parameter μ?

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*A1 – 6. Suppose drivers scoring 90 or more on their driver's test are labelled 'good' (G), and those scoring below 90 are labelled 'bad' (B). The table at the right gives statistics on the involvement of 'good' and 'bad' drivers in two-car accidents. Also suppose the courts always find one driver at fault in such accidents, and they revoke a driver's licence immediately after he or she has been found at fault in three two-car accidents (other kinds of accidents are ignored). Find the probability a driver's licence will be revoked immediately after his or her *fifth* two-car accident:

(a) for *good* drivers; (b) for *bad* drivers.

Driver at fault	Driver blameless	Number of accidents
G	G	5,000
G	B	15,000
B	G	25,000
B	B	55,000

*Optional more challenging problem for enrichment