6. On a particular stretch of highway, the speed of each vehicle is measured by radar at a speed trap. However, the radar measurements are not always accurate – they correctly indicate 95% of those speeding but wrongly indicate speeding for 10% of those driving within the speed limit. If 30% of vehicles on the highway are speeding, find the percentage of vehicles caught in the speed trap which are *actually* speeding.

This question involves **conditional probability.**

We can use the following tree diagram to organize the information given in the statement of the question, where:

event S is *vehicle is speeding* and ^cS is *vehicle is* not *speeding* (the complement of S),
event R is *radar indicates vehicle is speeding*;

and: $\Pr(S) = 0.3,$ $\Pr(R|S) = 0.95,$
         $\Pr(^cS) = 0.7,$ $\Pr(R|^cS) = 0.1.$
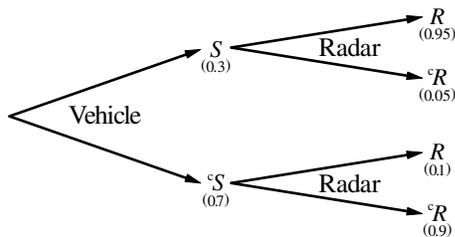
$$\boxed{0.8028 \simeq 80\%}$$
Percentage

$$\Pr(R) = \Pr(R|S) \times \Pr(S) + \Pr(R|^cS) \times \Pr(^cS)$$
$$= 0.95 \times 0.3 + 0.1 \times 0.7$$
$$= 0.285 + 0.07 = 0.355;$$

$$\Pr(^cS|R) = \frac{\Pr(R|^cS) \times \Pr(^cS)}{\Pr(R)} \qquad \text{[Bayes' rule]}$$

$$= \frac{0.285}{0.355} = 0.8028 \simeq 80\%;$$

i.e., around 80% of vehicles caught in the speed trap are *actually* speeding