MARKS

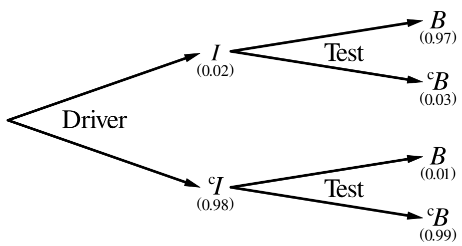
7

(6, 1)

6. Suppose that 2% of all drivers on the road on Sundays are intoxicated, and that 97% of those who are intoxicated are detected by a breathalyzer test. Suppose also that, due to imperfections in the breathalyzer, 1% of drivers who are **not** intoxicated register as being intoxicated.
   (a) If a driver on a Sunday registers as being intoxicated on a breathalyzer test, find the probability the driver is **not** intoxicated.
   (b) What do you conclude from your probability in (a) about the reliability required in a test for a relatively rare but important condition? Explain briefly.

(a) We can use the following tree diagram to organize the information given in the statement of the question, where:
event I is *driver is intoxicated* and ${}^{c}I$ is *driver is* not *intoxicated* (the complement of I),
event B is *breathalyzer test indicates intoxication*.

0.3356	(a)
Probability	



$$\Pr(B) = \Pr(B|I)\times\Pr(I) + \Pr(B|{}^{c}I)\times\Pr({}^{c}I)$$
$$= 0.97\times 0.02 + 0.01\times 0.98$$
$$= 0.0194 + 0.0098 = 0.0292;$$

$$\Pr({}^{c}I|B) = \frac{\Pr(B|{}^{c}I)\times\Pr({}^{c}I)}{\Pr(B)} \qquad \text{[Bayes' rule]}$$
$$= \frac{0.0098}{0.0292} = 0.3356 \simeq 34\%.$$

Thus, **over one-third** (about 34%) of drivers on a Sunday who test positive with the breathalyzer are **not** intoxicated (based on the hypothetical values in this question); this is clearly an **un**acceptably high error rate for the breathalyzer test.

(b) Looking at the expression above for $\Pr(B)$, we see that the high error rate could be reduced if $\Pr(B|{}^{c}I)$ were to be decreased; that is, the **false positive** rate of 1% for the breathalyzer test needs to be substantiall **reduced** in a situation like this.