MARKS

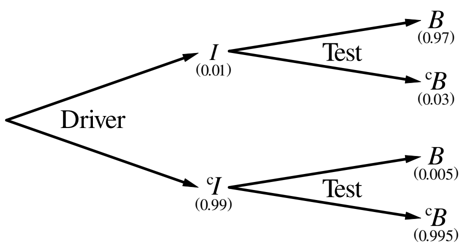
7

(6, 1)

6. Suppose that 1% of all drivers on the road on Sundays are intoxicated, and also suppose that a breathalyzer correctly indicates intoxication for 97% of all intoxicated drivers who are tested.
   (a) If the breathalyzer *in*correctly indicates intoxication for 0.5% of drivers who are tested but are *not* intoxicated, find the probability that a driver whose breathalyzer test is positive really is intoxicated.
   (b) Describe briefly what your calculations in (a) indicate about the accuracy required for the reading given by a breathalyzer.

0.6621	(a)
Probability	

(a) We can use the following tree diagram to organize the information given in the statement of the question, where:
event I is *driver is intoxicated* and ${}^{c}I$ is *driver is* not *intoxicated* (the complement of I),
event B is *breathalyzer test indicates intoxication*.



$$\Pr(B) = \Pr(B|I)\times\Pr(I) + \Pr(B|{}^{c}I)\times\Pr({}^{c}I)$$
$$= 0.97\times 0.01 + 0.005\times 0.99$$
$$= 0.0097 + 0.00495 = 0.01465;$$

$$\Pr(I|B) = \frac{\Pr(B|I)\times\Pr(I)}{\Pr(B)} \qquad \text{[Bayes' rule]}$$
$$= \frac{0.0097}{0.01465} = 0.6621 \simeq 66\%.$$

Thus, **fewer than two-thirds** (about 66%) of drivers on a Sunday who test positive with the breathalyzer **are** intoxicated (based on the hypothetical values in this question); this is clearly an **un**acceptably high error rate for the breathalyzer test.

(b) Looking at the expression above for $\Pr(B)$, we see that the high error rate could be reduced if $\Pr(B|{}^{c}I)$ were to be decreased; that is, the **false positive** rate of 0.5% for the breathalyzer test needs to be substantially **reduced** in a situation like this.