MARKS
7
(5, 2)

6. Three urns contain the numbers of balls of three different colours shown at the right below. An urn is selected equiprobably ('at random') and a ball is chosen equiprobably from it.

Urn *A*:	4 red;	2 white;	2 black
Urn *B*:	2 red;	1 white;	3 black
Urn *C*:	1 red;	3 white;	3 black

(a) Find the probability the ball is *white*.
(b) *If* the ball is white, find the probability urn *A* was the one selected.

(a) This problem involves **conditional probability.**

Define: event *A*: *ball is selected from urn A*,
B: *ball is selected from urn B*,
C: *ball is selected from urn C*,
W: *ball is white.*

Then: $\Pr(A) = \Pr(B) = \Pr(C) = \frac{1}{3}$ because the urn is selected **equiprobaby**,

and: $\Pr(W|A) = \frac{2}{8} = \frac{1}{4}$; $\Pr(W|B) = \frac{1}{6}$; $\Pr(W|C) = \frac{3}{7}$.

Hence:
$$\Pr(W) = \Pr(W|A)\times\Pr(A) + \Pr(W|B)\times\Pr(B) + \Pr(W|C)\times\Pr(C)$$
$$= \tfrac{1}{4}\times\tfrac{1}{3} + \tfrac{1}{6}\times\tfrac{1}{3} + \tfrac{3}{7}\times\tfrac{1}{3} = \tfrac{1}{12} + \tfrac{1}{18} + \tfrac{1}{7}$$
$$= \tfrac{21+14+36}{252} = \tfrac{71}{252} \simeq 0.2817.$$

$\boxed{\frac{71}{252} \simeq 0.2817}$ (a)
Probability

(b) Using Bayes' rule, we have:

$$\Pr(A|W) = \frac{\Pr(W|A)\times\Pr(A)}{\Pr(W)}$$
$$= \frac{\frac{1}{4}\times\frac{1}{3}}{\frac{71}{252}}$$
$$= \tfrac{21}{71} \simeq 0.2958.$$

$\boxed{\frac{21}{71} \simeq 0.2958}$ (b)
Probability