MARKS

5

8. A creamery produces a large number of packages of butter; it is known from experience that the weights in grams of the packages can be modelled by a normal distribution with a standard deviation of 5 grams. At what value should the average package weight be set so that at least 95% of the packages weigh at least 450 grams?

Let the random variable W represent the weight (in grams) of an equiprobably('randomly')-selected butter package;

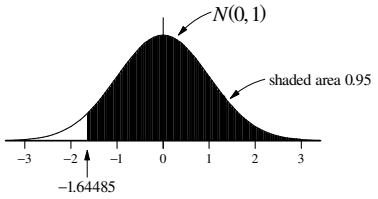
we use the model: $W \sim N(\mu, 5)$.

We want: $\Pr(W > 450) = 0.95$,

i.e., $\Pr\left(\frac{W-\mu}{\sigma} > \frac{450-\mu}{5}\right) = 0.95$ (standardizing),

i.e., $\Pr\left[N(0,1) > \frac{450-\mu}{5}\right) = 0.95$

$\therefore$ $\frac{450-\mu}{5} = -1.64485$ so that: $\mu = 458.2243 \simeq 458.2$ grams;



458.2 g

Average

i.e., the average package weight should be set at about 458.2 grams.

[The 'cost' of ensuring that 95% of packages contain 450 grams of butter is an average package weight with about 8.2 grams of 'free' butter; *process improvement* would entail reducing the standard deviation of the process that produces the butter packages.]