4. In a lottery, each ticket is filled out by the purchaser with six numbers chosen with*out* replacement from 1, 2, ..., 49. The six *winning* numbers are chosen at random without replacement, also from 1, 2, ..., 49; the *order* in which these six numbers are drawn is irrelevant.

MARKS

7

(2, 2, 3)

(a) To win first prize, a ticket must contain *all* the six numbers drawn; find the probability a person with one ticket wins first prize.

(b) If the lottery is conducted weekly, find the probability at least one of the six numbers drawn this week was also drawn last week.

(c) If 40 million tickets are sold for a particular draw of the lottery, find the probability there is a least one winner of first prize; indicate your assumption(s) clearly.

(a) The numbers are selected **without** replacement and order is *not* important, so we use **unordered** counting.

$$\Pr(1\text{st prize}) = \frac{\binom{6}{6}\binom{43}{0}}{\binom{49}{6}}$$ (*all 6* of the winning numbers, and *none* of the other 43, must be selected from the 49 possible numbers)

$$= \frac{1}{\binom{49}{6}} = \frac{1}{13{,}983{,}816}$$

$$\simeq 0.000\,000\,071\,511.$$

$$\frac{1}{\binom{49}{6}} = \frac{1}{13{,}983{,}816} \simeq 7.151\times10^{-8}$$

Probability (a)

(b) We approach this probability via the **complement** of the event of interest.

$$\Pr(\geq 1 \text{ number the same}) = 1 - \Pr(\text{all 6 numbers different this week})$$

$$= 1 - \frac{\binom{6}{0}\binom{43}{6}}{\binom{49}{6}}$$ (all 6 of the numbers this week must be selected from the 43 *not* chosen last week)

$$= 1 - \frac{\binom{43}{6}}{\binom{49}{6}} = \frac{563{,}383}{998{,}844}$$

$$\simeq 0.564\,035.$$

$$1 - \frac{\binom{43}{6}}{\binom{49}{6}} = \frac{563{,}383}{998{,}844} \simeq 0.5640$$

Probability (b)

(c) We also approach this probability via the **complement** of the event of interest.

For an individual ticket: $\Pr(\text{not a winner}) = \frac{13{,}983{,}815}{13{,}983{,}816}$, so that:

$$\Pr(\geq 1 \text{ winner in } 40{,}000{,}000 \text{ tickets}) = 1 - \Pr(\text{no winners in } 40{,}000{,}000 \text{ tickets})$$

$$= 1 - \left(\frac{13{,}983{,}815}{13{,}983{,}816}\right)^{40{,}000{,}000}$$ (assuming ticket numbers are chosen **independently**)

$$\simeq 0.942\,756 \simeq 94\%.$$

$0.9428 \simeq 94\%$

Probability (c)