MARKS
7
(2, 3, 2)

4. The numbers 1, 2, ..., 9 are written on nine cards and placed in a hat; the cards are then drawn one by one by equiprobable selecting ('at random') without replacement. Find the probability of each of the following events:
   (a) A: "exactly three odd numbers are obtained in the first five draws";
   (b) B: "exactly five draws are required to get three odd numbers";
   i.e., there are two odd numbers in the first four draws, followed by an odd number on the fifth draw;
   (c) C: "the largest number obtained in the first five draws is 5."

(a) The numbers are selected **without** replacement and order is *not* important, so we use **unordered** counting.

The number of possible selections of five cards is $\binom{9}{5} = \frac{9\times8\times7\times6\times5}{1\times2\times3\times4\times5} = 126$.

$$\Pr(\text{exactly three odd numbers in first five draws}) = \frac{\binom{5}{3}\binom{4}{2}}{\binom{9}{5}}$$ (select 3 of 5 *odd* numbers and 2 of 4 *even* numbers to satisfy the condition of 3 odd numbers in 5 draws)

$$= \frac{10\times6}{126} = \frac{10}{21} \simeq 0.476\,1905.$$

(A) $\boxed{\frac{10}{21} \simeq 0.4762}$ (a)
Probability

(b) $$\Pr(\text{exactly five draws to get three odd numbers}) = \frac{\binom{5}{2}\binom{4}{2}}{\binom{9}{5}}\times\frac{3}{5}$$ (select 2 of 5 *odd* numbers and 2 of 4 *even* numbers and then an odd number from remaining 3 odd and 2 even numbers)

$$= \frac{10\times6}{126}\times\frac{3}{5} = \frac{2}{7} \simeq 0.285\,7143$$

(B) $\boxed{\frac{2}{7} \simeq 0.2857}$ (b)
Probability

(c) $$\Pr(\text{largest number in first five draws is 5}) = \frac{\binom{1}{1}\binom{2}{2}\binom{2}{2}}{\binom{9}{5}}$$ (select 5 and then the *odd* numbers 1 and 3 and the *even* numbers 2 and 4 to satisfy the condition of 5 being the largest number)

$$= \frac{1\times1\times1}{126} = \frac{1}{126} \simeq 0.007\,936\,508.$$

(C) $\boxed{\frac{1}{126} \simeq 0.007\,937}$ (c)
Probability

The answer from the formal calculation of the probability in (c) reminds us of the more intutiive recognition that there is only *one* selection – with the *smallest* five numbers 1, 2, 3, 4, 5 – that satisfies the conditions of (c).