MARKS

10

(3, 4, 3)

7. The Chapin Social Insight Test is designed to evaluate how accurately the person doing the Test appraises other people. In the reference population used to develop the Test, suppose that Test scores can be modelled by a normal distribution with a mean of 25 and a standard deviation of 5; the range of possible scores is 0-41.

(a) Find the proportion of the reference population with Test scores of 20 and greater.

(b) Find the proportion of the reference population with Test scores between 10 and 20. Explain briefly if it matters whether the end points (10 and 20) of this interval are *in*cluded or *ex*cluded from the probability calculation.

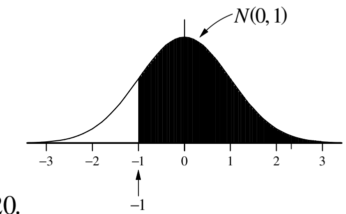
(c) How high a Test score must be obtained to be in the top quarter of the reference population in social insight?

(a) Let the random variable Y represent the Test score of an individual selected equiprobably ('at random') from the reference population;

we use the model: $Y \sim N(25, 5)$.

Then:
$$\begin{aligned}\Pr(Y \geq 20) &= \Pr[N(25, 5) \geq 20] \\ &= \Pr[N(0,1) \geq \tfrac{20-25}{5}] \quad \text{(standardizing)} \\ &= \Pr[N(0,1) \geq -1] \\ &= 0.8413;\end{aligned}$$

i.e., about 84% of the reference population has a test score of at least 20.



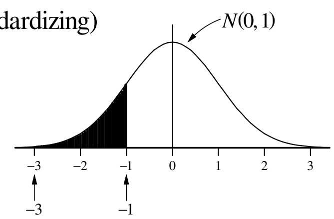
0.8413	(a)

Proportion

(b) Following the same reasoning as in (a):

$$\begin{aligned}\Pr(10 \leq Y \leq 20) &= \Pr[10 \leq N(25, 5) \leq 20] \\ &= \Pr[\tfrac{10-25}{5} \leq N(0,1) \leq \tfrac{20-25}{5}] \quad \text{(standardizing)} \\ &= \Pr[-3 \leq N(0,1) \leq -1] \\ &= 0.998650 - 0.84134 = 0.15731;\end{aligned}$$

i.e., about 16% of the reference population has a test score of between 10 and 20.



0.1573	(b)

Proportion

It does *not* matter whether the interval end points are *in*cluded or *ex*cluded; probability (or proportion) is represented by *area* under the normal p.d.f. and there is *zero* area associated with any *individual* value such as 10 or 20.
[This reminds us that the properties of models are *not* necessarily the same as those of the real world.]

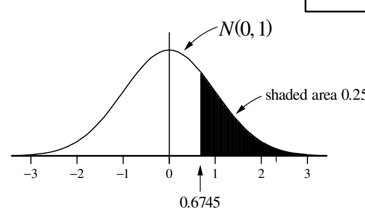
(c) The top quarter of the reference population lies above the 75*th* percentile of the distribution of scores – the shaded area in the diagram at the right.

From the $N(0,1)$ table, the 75*th* percentile of the $N(0,1)$ is 0.6745,

so the 75*th* percentile of the $N(25, 5)$ distribution is:

$$25 + 0.6745 \times 5 = 28.3725 \simeq 28.4;$$

i.e., a Test score of about 28.4 or greater must be obtained to lie in the top quarter of the reference population.



28.4	(c)

Score