7. Suppose that the IQ scores of the people in a population can be modelled by a normal distribution with a mean of 100 and a standard deviation of 15.

MARKS

10

(2, 5, 3)

(a) Find the proportion of the population with IQ scores of 130 and greater.

(b) Find the proportion of the population with IQ scores between 90 and 120. Explain briefly if it matters whether the end points (90 and 120) of this interval are *in*cluded or *ex*cluded from the probability calculation.

(c) Find the IQ score that represents the *99*th percentile of the distribution of IQ scores in this population.

(a) Let the random variable Y represent the IQ score (in points) of a person selected equiprobably ('at random') from the population;

we use the model: $Y \sim N(100, 15)$.

We want: $\Pr(Y \geq 130) = \Pr\left(\frac{Y-\mu}{\sigma} \geq \frac{130-100}{15}\right)$ (standardizing),

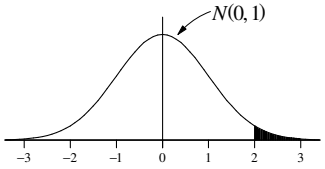
$= \Pr[N(0,1) \geq 2]$

$= 1 - 0.9772 = 0.0228 \simeq 0.023;$

i.e., about 2.3% of the population have IQ scores of 130 and greater.

2.3% (a)

Proportion



(b) We want: $\Pr(90 < Y \leq 120) = \Pr\left(\frac{90-100}{15} < \frac{Y-\mu}{\sigma} \leq \frac{120-100}{15}\right)$ (standardizing),

$= \Pr[-0.\dot{6} < N(0,1) \leq 1.\dot{3}]$

$= 0.24750 + 0.40879$

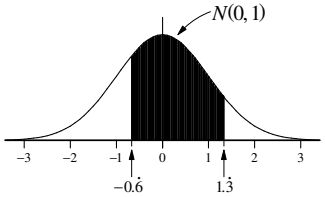
$= 0.65629 \simeq 0.66;$

i.e., about 66% of the population have IQ scores between 90 and 120.

Probability is represented by **area** under a p.d.f.; because there is no area associated with either end point of an interval (or with **any** individual value of a continuous randon variable), it does **not** matter whether the end points are **in**cluded or **ex**cluded from the calculation.

66% (b)

Proportion



(c) Let c be the required *99*th percentile of IQ scores.

We want: $\Pr(Y \leq c) = 0.99,$

i.e., $\Pr\left(\frac{Y-\mu}{\sigma} \leq \frac{c-100}{15}\right) = 0.99$ (standardizing),

i.e., $\Pr\left[N(0,1) \leq \frac{c-100}{15}\right] = 0.99;$

$\therefore \quad \frac{c-100}{15} = 2.3263$ so that: $c = 134.89 \simeq 135;$

i.e., the *99*th percentile of IQ scores is about 135.

135 (c)

*99*th percentile