3. The number of flaws per square metre in a type of carpet material varies with average 1.6 and standard deviation 1.2 flaws per square metre. The distribution of the number of flaws per square metre is not normal – in fact, it is discrete. An inspector studies 200 square metres of the carpet, records the number of flaws in each square metre, and then calculates the *average* number of flaws per square metre.

MARKS

7 (2, 5)

(a) Name the theorem that provides the basis for assuming approximate normality of the distribution of the average number of flaws per square metre of the carpet material; also indicate briefly the factor(s) which influence how accurate the approximation will be.

(b) Find the approximate probability that the average number of flaws per square metre exceeds 1.8.

(a) The name is the **Central Limit Theorem**.

The *accuracy* of the approximate normality of a sum or average depends on two factors:

- the value of n – the *larger* the value, the *better* the approximation;
- the shape of the distribution(s) of the Y_js – the *more symmetrical* they are, the *better* the approximation.

(b) Let the random variable Y_j represent the number of flaws in an equiprobably('randomly')-selected square metre of carpet (j = 1, 2, ... 200);

0.0092 ≃ 1%

Probability (b)

we are told in the question that $E(Y_j) = 1.6$ flaws, $s.d.(Y_j) = 1.2$ flaws.

The average number of flaws per square metre in 200 square metres of carpet is represented by the random variable $\overline{Y}$, where: $\overline{Y} = \frac{1}{200}\sum_{j=1}^{200} Y_j$

so that: $E(\overline{Y}) = E[\frac{1}{200}\sum_{j=1}^{200} Y_j] = \frac{1}{200}\sum_{j=1}^{200} E(Y_j) = \frac{1}{200}[1.6 + ... + 1.6] = 1.6$ flaws;

$$s.d.(\overline{Y}) = \frac{1}{200}\sqrt{\sum_{j=1}^{200}[s.d.(Y_j)]^2} = \frac{1}{200}\sqrt{[1.2^2 + ... + 1.2^2]} = 1.2/\sqrt{200} \simeq 0.084\,852\,813 \text{ flaws.}$$

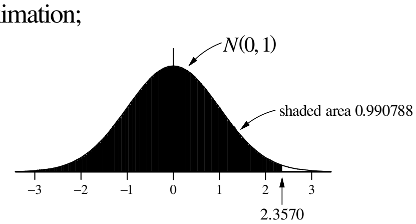
We hope that the (discrete) distribution of flaws per square metre is symmetrical enough that 200 terms in the average will be enough for a reasonably accurate normal approximation;

thus, we use the (approximate) model: $\overline{Y} \div N(1.6, 0.08485)$.

We want: $\Pr(\overline{Y} > 1.8) = \Pr(\frac{\overline{Y} - \mu}{\sigma} > \frac{1.8 - 1.6}{0.08485})$ (standardizing),

$= \Pr[N(0,1) > 2.3570]$

$= 1 - 0.990788 = 0.009212 \simeq 0.0092$;



i.e., the approximate probability of an average of more than 1.8 flaws per square metre is about 0.0092.