3. The distribution of the returns on common stocks is roughly symmetric but extreme observations are more frequent than in a normal distribution. Because the distribution is not strongly non-normal, the average return over even a moderate number of years is close to normal; in the long run, annual real returns on common stocks have varied with an average of about 9% and standard deviation about 28%. Judy plans to retire in 45 years, and is considering investing in stocks.

MARKS

8 (2, 5)

(a) Name the theorem that provides the basis for assuming approximate normality of the distribution of the average annual return on common stocks; also indicate briefly the factor(s) which influence how accurate the approximation will be.

(b) Assuming that the past pattern of variation continues, find the approximate probability that the average annual return on common stocks over the next 45 years will exceed 15%.

(a) The name is the **Central Limit Theorem**.

The *accuracy* of the approximate normality of a sum or average depends on two factors:

- the value of n – the *larger* the value, the *better* the approximation;
- the shape of the distribution(s) of the Y_js – the *more symmetrical* they are, the *better* the approximation.

(b) Let the random variable Y_j represent the return on common stocks for an equiprobably('randomly')-selected year (j = 1, 2, ... 45);

$0.075 \simeq 7\frac{1}{2}\%$	(b)
Probability	

we are told in the question that $E(Y_j)$ = 9% = 0.09, $s.d.(Y_j)$ = 28% = 0.28.

The average return on common stocks over 45 years is represented by the random variable $\overline{Y}$, where:

$$\overline{Y} = \frac{1}{45}\sum_{j=1}^{45} Y_j$$

so that:

$$E(\overline{Y}) = E\left[\frac{1}{45}\sum_{j=1}^{45} Y_j\right] = \frac{1}{45}\sum_{j=1}^{45} E(Y_j) = \frac{1}{45}\left[0.09 + \ldots + 0.09\right] = 0.09 = 9\%;$$

$$s.d.(\overline{Y}) = \frac{1}{45}\sqrt{\sum_{j=1}^{45}[s.d.(Y_j)]^2} = \frac{1}{45}\sqrt{\left[0.28^2 + \ldots + 0.28^2\right]} = 0.28/\sqrt{45} \simeq 0.041\,739\,935 \simeq 4.173\,9935\%.$$

We hope that the (discrete) distribution of returns on common stocks is symmetrical enough that 45 terms in the average will be enough for a reasonably accurate normal approximation;

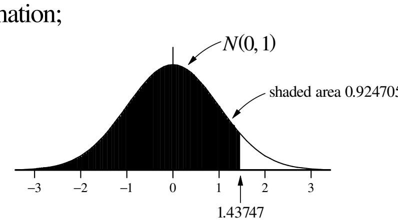
thus, we use the (approximate) model: $Y \dot{\sim} N(0.09,\ 0.04174)$.

We want:

$$\Pr(Y > 0.15) = \Pr\left(\frac{Y-\mu}{\sigma} > \frac{0.15 - 0.09}{0.04174}\right) \quad \text{(standardizing)},$$

$$= \Pr\left[N(0,1) > 1.43747\right]$$

$$= 1 - 0.924705 = 0.075295 \simeq 0.075;$$



i.e., the approximate probability of an average return over 45 years of more than 15% is about 0.075.