MARKS

6
(4, 2)

3. My neighbour and I have identical floodlamps with lifetimes that can be modelled by an exponential distribution with mean $\theta = 400$ hours. If we both burn our lamps for 5 hours each night, find the probability that:
(a) my floodlamp lasts longer than 100 nights;
(b) both floodlamps last longer than 100 nights.

(a) Let the random variable T represent the lifetime of an equiprobably('randomly')-selected floodlamp; we are told that the lifteimes can be modelled by an exponential distribution with mean $\theta = 400$ hours, so that:

$$T \sim Exp(\theta = 400) \quad \text{where: } f(t) = \tfrac{1}{400} e^{-t/400} \quad (t > 0).$$

Then, because 100 nights involves 500 hours of use:

$$\begin{aligned} \Pr(T > 500) &= \int_{500}^{\infty} \tfrac{1}{400} e^{-t/400} dt \\ &= -e^{-t/400} \Big|_{500}^{\infty} \\ &= 0 + e^{-500/400} \\ &= e^{-5/4} \\ &\simeq 0.2865 \simeq 29\%; \end{aligned}$$

i.e., the required probability is 0.2865 or about 29%.

0.2865	(a)
Probability	

(b) Assuming the lifetimes of the two floodlamps are **probabilistically independent**, we can **multiply** the individual probabilities found in (a) to obtain the required probability of the **intersection** of two events;

hence:

$$\begin{aligned} \Pr(\text{both floodlamps last longer than 100 nights}) &= [\Pr(T > 500)]^2 \\ &= [e^{-5/4}]^2 \\ &= e^{-5/2} \\ &\simeq 0.08208 \simeq 8.2\%; \end{aligned}$$

i.e., the required probability is 0.08208 or about 8.2%.

0.08208	(b)
Probability	