9. The specifications of corks for wine bottles call for corks whose diameters are between 2.90 and 3.10 cm. A cork which does not meet these specifications is classified as defective; if it is too small, it leaks and causes the wine to deteriorate, whereas a cork that is too large will not fit in the bottle.

MARKS

6 Suppose there are two cork cutting machines; **Machine 1** produces corks whose diameters can be modelled by a normal distribution with mean 3.00 cm and standard deviation 0.05 cm, while **Machine 2** produces corks whose diameters can also be modelled by a normal distribution but with mean 3.05 cm and standard deviation 0.04 cm. Which machine would you recommend based on the anticipated proportion of defects? (Your answer needs to include explicit calculation of relevant normal probabilities.)

1
Machine number

Let the random variable D_i represent cork diameter (in cm) from Machine i, $i = 1, 2$;
we use the models: $D_1 \sim N(3.00, 0.05)$; $D_2 \sim N(3.05, 0.04)$.

We calculate the proportions of *acceptable* corks from the two machines as:

$$\begin{aligned} &\Pr(2.90 \le D_1 \le 3.10) && \Pr(2.90 \le D_2 \le 3.10) \\ &= \Pr\left(\frac{2.90-3.00}{0.05} \le \frac{D_1-\mu_1}{\sigma_1} \le \frac{3.10-3.00}{0.05}\right) && = \Pr\left(\frac{2.90-3.05}{0.04} \le \frac{D_2-\mu_2}{\sigma_2} \le \frac{3.10-3.05}{0.04}\right) \quad \text{(standardizing)} \\ &= \Pr[-2 \le N(0,1) \le 2] && = \Pr[-3.75 \le N(0,1) \le 1.25] \\ &= 2\times(0.97725-0.5) && = (0.9999\,1158-0.5)+(0.89435-0.5) \\ &= 0.95450 \text{ (or 4.550\% defective);} && = 0.89426 \text{ (or 10.574\% defective).} \end{aligned}$$

Hence, on the basis of the normal models, we recommend **Machine 1** (with a defective rate of about 4.5%) over Machine 2 (with a defective rate of about 10.6%, more than **twice** as high).

NOTE: In the real world (as discussed in Part 11 of the STAT 221 Course Materials), these defect rates would need to be *at least* one order of magnitude lower in any adequate cork manufacturing process.