MARKS

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1. In a factory which manufactures beam balances from components, each balance is assembled by attaching a randomly-selected pan and pan-holder to each end of the balance arm. The distribution of pan weights in grams can be modelled by a $N(50, \sqrt{0.0005})$ distribution, and the model for the weights of the pan-holders is $N(10, \sqrt{0.0003})$. A balance is *un*acceptable if the combined weights of the pan and pan-holder on each side of the balance differ by more than 0.080 gm. What proportion of the balances manufactured in the factory will be unacceptable?

$0.0455 \simeq 4½\%$
Proportion

This question involves **linear combinations** of probabilistically independent normally distributed random variables.

Let the random variable P represent the weight (in grams) of a randomly-selected pan, and the random variable H represent the weight (in grams) of a randomly-selected pan holder;

We use the models: $P \sim N(50, \sqrt{0.0005})$ and: $H \sim N(10, \sqrt{0.0003})$,

so that: $P + H \sim N(60, \sqrt{0.0008})$.

Then, for the difference in weight between two sides of a balance:

$$D = (P_1 + H_1) - (P_2 + H_2) \sim N(0, 0.04).$$

Hence: $\Pr(-0.08 \le D \le 0.08)$

$= \Pr[-2 \le N(0, 1) \le 2]$ (standardizing)

$= 2 \times (0.97725 - 0.5) = 0.95450$; [from the $N(0, 1)$ table];

thus, the proportion of **un**acceptable balances is:

$$1 - 0.95450 = 0.04550 \simeq 4½\%.$$