MARKS

7

(2, 5)

1. The clearance between a pin and the collar around it is important for the reliable operation of a disc drive for personal computers. Specifications call for the pin to have external diameter 0.525 cm and the collar to have internal diameter 0.526 cm; the clearance will then be 0.001 cm. In practice, both diameters vary from part to part independently of each other. The external diameter, E, of the pin can be modelled by a $N(0.525, 0.0003)$ distribution and the collar internal diameter, I, can be modelled by a $N(0.526, 0.0004)$ distribution.
   (a) Explain briefly why it is reasonable to model the two diameters as being *probabilistically independent.*
   (b) Find the probability the pin will *not* fit inside the collar.

(a) The pin and the collar are almost certainly manufactured in separate processes, possibly by different companies in different locations. There is thus no obvious way in which the diameter of a pin can influence that of a collar, and *vice versa*; hence, it *is* reasonable to assume that the two diameters can reasonasbly be modelled as being probabilistically independent.

(b) Let the random variable E represent pin external diameter (in cm), and the random variable I represent collar internal diameter (in cm);

2.3%	(b)
Probability	

the question directs us to use the models: $E \sim N(0.525, 0.0003)$, $I \sim N(0.526, 0.0004)$.

A pin will *not* fit inside a collar when $E > I$ or $E - I > 0$.

For the random variable $E - I$, we can use the model: $E - I \sim N(-0.001, 0.0005)$;
[when dealing with the *difference* of two independent normal random variables, the **distribution** remains normal, the **mean** is the difference of the two individual means, and the **standard deviation** is the square root of the *sum* of the squares of the two individual standard deviations].

Then:

$$\begin{aligned}\Pr(E - I > 0) &= \Pr[N(-0.001, 0.0005) > 0] \\ &= \Pr[N(0,1) > \tfrac{0-(-0.001}{0.0005}] \qquad \text{(standardizing)} \\ &= \Pr[N(0,1) > 2] \\ &= 0.0228 \simeq 2.3\%;\end{aligned}$$

i.e., the probability the pin will *not* fit inside the collar is about 2.3%.