2. A continuous random variable Y has the probability density function given at the right.

$$f(y) = \begin{cases} ky^{-2} & ; \quad y \ge 1, \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

MARKS

7 (3, 2, 2)

(a) Evaluate the constant k and sketch the p.d.f. of Y.
(b) Evaluate: $\Pr(2 < Y \le 4)$.
(c) Find the median of Y.

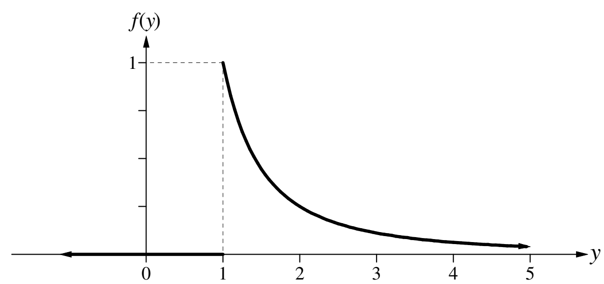
(a) Using the normalizing condition, we have: $\int_{-\infty}^{1} 0\,dy + k\int_{1}^{\infty} y^{-2}dy = 0 - \frac{k}{y}\Big|_{1}^{\infty} = -0 + k = 1,$

1	(a)
k	

so that: $k = 1$ and the p.d.f. of Y is:

$$f(y) = \begin{cases} y^{-2} & ; \quad y \ge 1, \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

A sketch of the p.d.f. is shown at the right.



(b) $\Pr(2 < Y \le 4) = \int_{2}^{4} f(y)dy = \int_{2}^{4} y^{-2}dy = -\frac{1}{y}\Big|_{2}^{4} = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4} = 0.25.$

¼ = 0.25	(b)
Probability	

2	(c)
Median	

(c) At the median (m, say), the area under the p.d.f. is **half** that under the whole curve;

i.e., $0.5 = \int_{-\infty}^{m} f(y)dy = \int_{-\infty}^{1} 0\,dy + \int_{1}^{m} y^{-2}dy = 0 - \frac{1}{y}\Big|_{1}^{m} = -\frac{1}{m} + 1;$

hence: $-\frac{1}{2} = -\frac{1}{m}$ so the median is m = 2.