MARKS

6 (4, 2)

3. The two (identical and independent) channels of my stereo amplifier have lifetimes that can each be modelled by an exponential distribution with mean $\theta = 2{,}500$ hours. If I play my stereo for 2 hours every day, find the probability that:
(a) the channel operating the *right*-hand loud speaker lasts longer than five (365-day) years;
(b) the channels *both* last longer than five (365-day) years.

(a) This problem involves the **exponential distribution** as a continuous probability model.

$e^{-1.46} \simeq 0.2322$ (a)
Probability

Let the random variable L represent channel lifetime (in hours);
we use the model: $L \sim Exp(\theta = 2{,}500)$.

Now 2 hours per day for 5 years is $2 \times 365 \times 5 = 3{,}650$ hours;

hence:

$$\begin{aligned}\Pr(L > 3{,}650) &= \int_{3{,}650}^{\infty} f(l)\,dl \\ &= \frac{1}{2{,}500}\int_{3{,}650}^{\infty} e^{-l/2{,}500}\,dl \\ &= -e^{-l/2{,}500}\Big|_{3{,}650}^{\infty} \\ &= -0 + e^{-3{,}650/2{,}500} = e^{-1.46} \simeq 0.232\,236,\end{aligned}$$

which is the required probability the channel operating the right-hand loud speaker lasts longer than five years.

(b) The probability for each channel *separately* is that found in (a), because we are told the channels are **identical**.

$e^{-2.92} \simeq 0.0539$ (b)
Probability

Because we are also told that the channels are **independent**, we can find the probability for *both* channels (represented by the **intersection** of the relevant events) by **multiplying** the two individual probabilities from (a).

Hence:

$$\begin{aligned}\Pr(\text{both channels last longer than five years}) &= e^{-1.46} \times e^{-1.46} \\ &= e^{-2.92} \simeq 0.053\,934.\end{aligned}$$