MARKS
9
(3, 2, 2, 2)

2. A continuous random variable Y has the probability density function given at the right.

$$f(y) = \begin{cases} ky^{-3} & ; \quad y \geq 1, \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

(a) Evaluate the constant k and sketch the p.d.f. of Y.
(b) Evaluate: $\Pr(2 < Y \leq 4)$.
(c) Find the mean of Y.
(d) Find the median of Y.

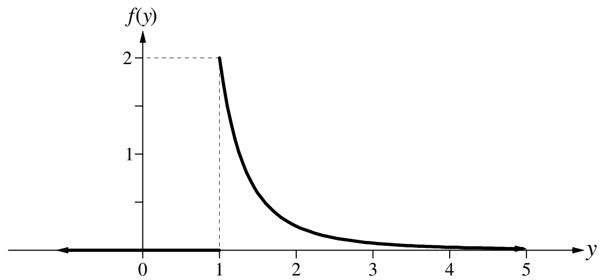
(a) Using the normalizing condition, we have: $\int_{-\infty}^{\infty} f(y)dy = \int_{-\infty}^{1} 0\,dy + k\int_{1}^{\infty} y^{-3}dy = 0 - \frac{k}{2y^2}\Big|_1^{\infty} = -0 + \frac{k}{2} = 1,$

2	(a)
k	

so that: $k = 2$ and the p.d.f. of Y is:

$$f(y) = \begin{cases} 2y^{-3} & ; \quad y \geq 1, \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

A sketch of the p.d.f. is shown at the right.



(b) To find the probability, we **integrate** the p.d.f.

$$\Pr(2 < Y \leq 4) = \int_{2}^{4} f(y)dy = \int_{2}^{4} 2y^{-3}dy = -\frac{1}{y^2}\Big|_2^4 = -\frac{1}{16} + \frac{1}{4} = \frac{3}{16} = 0.1875.$$

$\frac{3}{16} = 0.1875$	(b)
Probability	

(c) The **mean** of Y is:

$$E(Y) \equiv \mu_Y = \int_{-\infty}^{\infty} y \cdot f(y)dy = \int_{-\infty}^{1} y \cdot 0\,dy + \int_{1}^{\infty} y \cdot 2y^{-3}dy = 2\int_{1}^{\infty} y^{-2}dy = -\frac{2}{y}\Big|_1^{\infty} = -0 + 2 = 2.$$

2	(c)
Mean	

(d) At the **median** (m, say), the area under the p.d.f. is **half** that under the whole curve;

i.e., $0.5 = \int_{-\infty}^{m} f(y)dy = \int_{-\infty}^{1} 0\,dy + \int_{1}^{m} 2y^{-3}dy = 0 - \frac{1}{y^2}\Big|_1^{m} = -\frac{1}{m^2} + 1;$

hence: $-\frac{1}{2} = -\frac{1}{m^2}$ so the median is $m = \sqrt{2} \simeq 1.4142$.

$\sqrt{2} \simeq 1.4142$	(d)
Median	